SERIES

Powers of Natural Numbers

$$\sum_{k=1}^{n} k = \frac{1}{2}\, n(n+1)\ ;\ \ \sum_{k=1}^{n} k^2 = \frac{1}{6}\, n(n+1)(2n+1); \sum_{k=1}^{n} k^3 = \frac{1}{4}\, n^2(n+1)^2$$

Arithmetic $$S_n = \sum_{k=0}^{n-1}(a+kd) = \frac{n}{2}\left\{2a+(n-1)d\right\}$$

Geometric (convergent for $-1 < r < 1$)

$$S_n = \sum_{k=0}^{n-1} ar^k = \frac{a(1-r^n)}{1-r}\ ,\ S_\infty = \frac{a}{1-r}$$

Binomial (convergent for $|x| < 1$)

$$(1+x)^n = 1 + nx + \binom{n}{2}x^2 + ... + \binom{n}{r}x^r + ...$$

where $\binom{n}{r} = \dfrac{n(n-1)(n-2)...(n-r+1)}{r!}$

Maclaurin series

$$f(x) = f(0) + xf'(0) + \frac{x^2}{2!}f''(0) + ... + \frac{x^k}{k!}f^{(k)}(0) + R_{k+1}$$

$$\text{where } R_{k+1} = \frac{x^{k+1}}{(k+1)!}f^{(k+1)}(\theta x),\quad 0 < \theta < 1$$

Taylor series

$$f(a+h) = f(a) + hf'(a) + \frac{h^2}{2!}f''(a) + ... + \frac{h^k}{k!}f^{(k)}(a) + R_{k+1}$$

$$\text{where } R_{k+1} = \frac{h^{k+1}}{(k+1)!}f^{(k+1)}(a+\theta h)\ ,\ 0 < \theta < 1.$$

OR

$$f(x) = f(x_0) + (x-x_0)f'(x_0) + \frac{(x-x_0)^2}{2!}f''(x_0) + ... + \frac{(x-x_0)^k}{k!}f^{(k)}(x_0) + R_{k+1}$$

$$\text{where } R_{k+1} = \frac{(x-x_0)^{k+1}}{(k+1)!}f^{(k+1)}(x_0 + (x-x_0)\theta),\quad 0 < \theta < 1$$

Special Power Series

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + ... + \frac{x^r}{r!} + ... \qquad \text{(all } x\text{)}$$

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + ... + \frac{(-1)^r x^{2r+1}}{(2r+1)!} + ... \qquad \text{(all } x\text{)}$$

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + ... + \frac{(-1)^r x^{2r}}{(2r)!} + ... \qquad \text{(all } x\text{)}$$

$$\tan x = x + \frac{x^3}{3} + \frac{2x^5}{15} + \frac{17x^7}{315} + ... \qquad (|x| < \tfrac{\pi}{2})$$

$$\sin^{-1} x = x + \frac{1}{2}\frac{x^3}{3} + \frac{1.3}{2.4}\frac{x^5}{5} + \frac{1.3.5}{2.4.6}\frac{x^7}{7} +$$

$$... + \frac{1.3.5....(2n-1)}{2.4.6....(2n)}\frac{x^{2n+1}}{2n+1} + ... \qquad (|x| < 1)$$

$$\tan^{-1} x = x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + ... + (-1)^n \frac{x^{2n+1}}{2n+1} + ... \qquad (|x| < 1)$$

$$\ell n(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + ... + (-1)^{n+1}\frac{x^n}{n} + ... \qquad (-1 < x \leq 1)$$

$$\sinh x = x + \frac{x^3}{3!} + \frac{x^5}{5!} + \frac{x^7}{7!} + ... + \frac{x^{2n+1}}{(2n+1)!} + ... \qquad \text{(all } x\text{)}$$

$$\cosh x = 1 + \frac{x^2}{2!} + \frac{x^4}{4!} + \frac{x^6}{6!} + ... + \frac{x^{2n}}{(2n)!} + ... \qquad \text{(all } x\text{)}$$

$$\tanh x = x - \frac{x^3}{3} + \frac{2x^5}{15} - \frac{17x^7}{315} + ... \qquad (|x| < \tfrac{\pi}{2})$$

$$\sinh^{-1} x = x - \frac{1}{2}\frac{x^3}{3} + \frac{1.3}{2.4}\frac{x^5}{5} - \frac{1.3.5}{2.4.6}\frac{x^7}{7} +$$

$$... + (-1)^n \frac{1.3.5...(2n-1)}{2.4.6...2n}\frac{x^{2n+1}}{2n+1} + ... \qquad (|x| < 1)$$

$$\tanh^{-1} x = x + \frac{x^3}{3} + \frac{x^5}{5} + \frac{x^7}{7} + ...\frac{x^{2n+1}}{2n+1} + ... \qquad (|x| < 1)$$