## MECHANICS

### Kinematics

Motion constant acceleration

$$\mathbf{v} = \mathbf{u} + \mathbf{f}t, \qquad \mathbf{s} = \mathbf{u}t + \frac{1}{2}\mathbf{f}t^2 = \frac{1}{2}(\mathbf{u} + \mathbf{v})t$$

$$\mathbf{v}^2 = \mathbf{u}^2 + 2\mathbf{f}.\mathbf{s}$$

General solution of $\frac{d^2x}{dt^2} = -\omega^2 x$ is

$$x = a\cos\omega t + b\sin\omega t = R\sin(\omega t + \phi)$$

where $R = \sqrt{a^2 + b^2}$ and $\cos\phi = a/R$, $\sin\phi = b/R$.

In polar coordinates the velocity is $(\dot{r}, r\dot{\theta}) = \dot{r}\mathbf{e}_r + r\dot{\theta}\mathbf{e}_\theta$ and the acceleration is $\left[\ddot{r} - r\dot{\theta}^2, r\ddot{\theta} + 2\dot{r}\dot{\theta}\right] = (\ddot{r} - r\dot{\theta}^2)\mathbf{e}_r + (r\ddot{\theta} + 2\dot{r}\dot{\theta})\mathbf{e}_\theta$.

### Centres of mass

The following results are for uniform bodies:

| | | |
|---|---|---|
| hemispherical shell, radius $r$ | $\frac{1}{2}r$ | from centre |
| hemisphere, radius $r$ | $\frac{3}{8}r$ | from centre |
| right circular cone, height $h$ | $\frac{3}{4}h$ | from vertex |
| arc, radius $r$ and angle $2\theta$ | $(r\sin\theta)/\theta$ | from centre |
| sector, radius $r$ and angle $2\theta$ | $(\frac{2}{3}r\sin\theta)/\theta$ | from centre |

### Moments of inertia

1. The moment of inertia of a body of mass $m$ about an axis $= I + mh^2$, where $I$ is the moment of inertial about the parallel axis through the mass-centre and $h$ is the distance between the axes.

2. If $I_1$ and $I_2$ are the moments of inertia of $a$ lamina about two perpendicular axes through a point 0 in its plane, then its moment of inertia about the axis through 0 perpendicular to its plane is $I_1 + I_2$.

3. The following moments of inertia are for uniform bodies about the axes stated:

| | |
|---|---|
| rod, length $\ell$, through mid-point, perpendicular to rod | $\frac{1}{12}m\ell^2$ |
| hoop, radius $r$, through centre, perpendicular to hoop | $mr^2$ |
| disc, radius $r$, through centre, perpendicular to disc | $\frac{1}{2}mr^2$ |
| sphere, radius $r$, diameter | $\frac{2}{5}mr^2$ |

### Work done

$$W = \int_{t_A}^{t_B} \mathbf{F}.\frac{d\mathbf{r}}{dt}dt.$$