ALGEBRAIC STRUCTURES

A group G is a set of elements $\{a, b, c, \dots\}$ — with a binary operation $*$ such that

1. $a * b$ is in G for all a, b in G
2. $a * (b * c) = (a * b) * c$ for all a, b, c in G
3. G contains an element e, called the identity element, such that $e * a = a = a * e$ for all a in G
4. given any a in G, there exists in G an element a^{-1}, called the element inverse to a, such that $a^{-1} * a = e = a * a^{-1}$.

A commutative (or Abelian) group is one for which $a * b = b * a$ for all a, b, in G.

A field F is a set of elements $\{a, b, c, \dots\}$ — with two binary operations $+$ and $.$ such that

1. F is a commutative group with respect to $+$ with identity 0
2. the non-zero elements of F form a commutative group with respect to $.$ with identity 1
3. $a.(b + c) = a.b + a.c$ for all a, b, c, in F.

A vector space V over a field F is a set of elements $\{\underline{a}, \underline{b}, \underline{c}, \dots\}$ — with a binary operation $+$ such that

1. they form a commutative group under $+$;

   and, for all λ, μ in F and all $\underline{a}, \underline{b}$, in V,
2. $\lambda\underline{a}$ is defined and is in V
3. $\lambda(\underline{a} + \underline{b}) = \lambda\underline{a} + \lambda\underline{b}$
4. $(\lambda + \mu)\underline{a} = \lambda\underline{a} + \mu\underline{a}$
5. $(\lambda.\mu)\underline{a} = \lambda(\mu\underline{a})$
6. if 1 is an element of F such that $1.\lambda = \lambda$ for all λ in F, then $1\underline{a} = \underline{a}$.

An equivalence relation R between the elements $\{a, b, c, \dots\}$ — of a set C is a relation such that, for all a, b, c in C

1. aRa (R is reflextive)
2. $aRb \Rightarrow bRa$ (R is symmetric)
3. (aRb and bRc) $\Rightarrow aRc$ (R is transitive).