Exam 1R1
Civil Engineering
Electrical Engineering
Systems Engineering

TWO HOURS

Answer Question 1 from Section A and two questions from Section B

Section A

1) a) Let A, B, C have coordinates (2,1,1), (-1,1,0), (-1,0,3) respectively. Find the position vector of A, the vectors $\underline{u} = \overrightarrow{AB}$ and $\underline{v} = \overrightarrow{AC}$ in the form $a\underline{i} + b\underline{j} + c\underline{k}$. Find the vectors

i) $2\underline{u} - 3\underline{v}$

ii) $\underline{u} \times \underline{v}$

b) The vector equation of a line L_1 is

$$\underline{r} = \begin{bmatrix} -1 \\ 3 \\ 0 \end{bmatrix} + \lambda \begin{bmatrix} 3 \\ -2 \\ 1 \end{bmatrix}$$

and the Cartesian equation of a line L_2 is

$$\frac{x}{2} = y = \frac{5 - z}{4}$$

Find the Cartesian equation of L_1 and the vector equation of the line L_2. Show that L_1 and L_2 meet at A and that the two lines are orthogonal.

Section B

2) a) Find the roots of the quadratic polynomial $x^2 + 3x + 2$

b) Show that the cubic polynomial $p(x)$

$$p(x) = x^3 + 5x^2 + 8x + 4$$

has a root at $x = -2$ and find a quadratic polynomial $q(x)$ such that

$$p(x) = (x + 2)q(x)$$

c) Using parts (a) and (b), express

$$F(x) = \frac{x^2 + 5x + 5}{x^3 + 5x^2 + 8x + 4}$$

as a partial fraction and hence evaluate $\int F(x)\, dx$

3) a) Sketch the function e^{-t} for for $-3 < t < 3$.

b) Differentiate with respect to x

i) $y(x) = \cos(3x^2 + 1) - x^{-5}$

ii) $y(x) = (\ln(x))^3$

c) By making the substitution $v(x) = \ln(x)$, or otherwise, find the area under the curve

$$\frac{(\ln(x))^2}{x}$$

for x in the range $1 < x < 2$.

d) By finding the first and second derivative of the function

$$y(x) = e^{-x^2}$$

show that $y(x)$ has a maximum at $x = 0$.

4) a) Sketch the function $\cos(x)$ in the range $-4\pi < x < 4\pi$ radians.

b) Find all values of θ, in degrees, such that

$$\cos(\theta) = 0.5$$

c) Find a value of M and an angle α, in radians, such that

$$2\cos(\theta) - 3\sin(\theta) = M\cos(\theta + \alpha)$$

and find all values of θ, in radians, such that

$$2\cos(\theta) - 3\sin(\theta) = -2$$

d) Use integration by parts to evaluate $\int_0^{\frac{\pi}{2}} \theta \ \sin(2\theta)\ d\theta$