Thoughts on how to choose a place to study for a first degree in maths in the U.K.

Bill Lionheart, School of Mathematics, University of Manchester

July 30, 2008

Where should a prospective undergraduate choose to study for a mathematics degree in the UK? It is an interesting question both from the perspective of potential students (who sometimes ask my advice) and from the point of view of an academic in a mathematics department trying to make their course and department better, and to recruit the students best suited to their course.

One difficulty faced by the potential maths student is a lack of informed and impartial advice. Teachers do not generally have an up-to-date view of the quality of maths courses. If the potential students do have access to academic mathematicians (and this is a minority before they have applied) they will have to account for an understandable bias to the institutions where they work or were trained. Then one is left with newspaper league tables, which from the point of view of insiders are generally laughable (more on this later), and of course the "market driven" approach that the course demanding the highest grades must be best.

My thoughts here are my personal perspective, and of course are biased towards the things I know. I am currently a professor of applied mathematics at the University of Manchester, I chose to go to Warwick for my undergraduate degree (inspired by the Royal Institution Christmas Lectures of Christopher Zeeman) and deliberately (perhaps stubbornly) not Cambridge (and I lived near Oxford so that was ruled out). I went on to study at the University of California Berkeley (again over Cambridge and Harvard), and then to study and work at Oxford Polytechnic. I collaborate widely with mathematicians, scientists and engineers from universities all over the world and from a wide range of industry.

I will focus at first on the able student who is keen on mathematics, and will certainly get an A grade in their A2 mathematics and a generally high grades on their other A-levels. Quite likely they will also take Further Maths at least to AS level. For the able maths enthusiast and extra A-level in mathematics is much easier than, say, Chemistry even if has to be done by distance learning [1] Such students will often be sufficiently serious about the subject to leave open the option of postgraduate study and a career in mathematics. In this case a research active department is essential. Unless the majority of the academic staff are engaged in research the curriculum will not reflect the latest trends and interesting new areas for research. Most importantly you should be looking for a vibrant research culture with plenty of research seminars and academic visitors. While as an undergraduate you will not go to many research seminars

[1] All students in England and Wales can now access the Further Mathematics Network's distance learning programme, as well as attending study sessions at regional centres

(although certainly I went to a few) these are signs of the vitality of the department. Also in many departments visitors and seminar speakers will occasionally give more accessible "colloquium" and popular talks as well.

From the perspective of an A-level student you will probably not have an over-view of the main areas of mathematics: roughly they are Logic & Foundations, Geometry & Topology, Algebra & Combinatorics and Analysis (i.e. calculus). The A-level idea of Pure, Applied and Statistics is largely misleading, in that A-level applied mathematics means "mechanics". Outside of the artificial culture of A-level Pure and Applied really are the perspective from which the mathematics is studied: pure "for it's own sake" and applied with a view to solving a problem from outside mathematics. In the UK academia there is a old and dying tradition that seems asymptotic methods in fluid dynamics as the only valid area of applied mathematics (and we were certainly very good at it!) but from a global point of view (and mathematics is a truly global enterprise) applied mathematics is any mathematics that gets applied! There is however a bias towards applications of analysis (i.e. calculus), as many physical systems are modelled by differential equations, however increasingly discrete mathematics is finding application abstract algebra and number theory for example are essential to cryptography and communications.

Many students are put off statistics at A-level and I believe this is still due to a kind of "recipe book" approach where you learn what test to apply but not the underlying mathematical analysis. It could be argued that this is relevant to those going on to study applied science, especially biology, medicine and psychology (although I am bound to say that they should be taught to use such recipes with great caution and in particular know when to consult a qualified statistician). However the the keen mathematics student this approach simply does not look like mathematics: we expect everything to be carefully justified. At university statistics is studied starting from the fundamental concepts of probability theory and the methods are derived using rigorous analysis. Courses with an inclination towards truly applied statistics will also teach you how to use these methods on real world problem. Like applied mathematics and mathematical modelling this has the additional difficulty of interacting with the non-mathematically trained, and one needs to be skilled in communication and interpretation before their problems and claims can be translated in to mathematics. Not everyone is interested in this interdisciplinary challenge, but if you think you might be it may be worth finding out if the course aims to teach this aspect, or if there are some of the academic staff who regularly work with people outside mathematics.

Almost certainly you will not know at this stage what area of mathematics you will choose as your specialism. Most degree courses have a fairly rigid system for the first one or two years with specialisation in the third (and for MMath fourth) years. This is important as it suggests that a department covering a wider range of mathematics will give you a broader view and a wider choice. It would seem something of a tragedy if young mathematician with a special gift for geometry or analysis chose to go to a small department that excelled in algebra and was consequently put off a career in mathematics. That said there are many merits to smaller institutions and departments. Many students are happier and perform better in a context where they know most of the staff and students and felt they belong and they are valued. At Manchester we merged two medium sized departments (Victoria University of Manchester and UMIST) and created the third largest (possibly the largest under one roof) mathematics department in the UK. Certainly some of the students who chose UMIST valued a medium sized department in small university (UMIST was essentially just science and technology). Students coming to Manchester now value the wide range of specialist options our size allows. We use small tutorial groups as well as a peer tution system where first years are mentored by more senior undergraduates to try to ensure everyone gets the help and support they need. Expect and look for such things to be in place in other large departments. Of course Oxford and Cambridge, the largest mathematics departments (especially if you take the union of the various pure applied and statistics parts), benefit from the college system, which ideally gives you the best of big and small. In this case people's experience varies widely between colleges, and choosing a college at Cambridge and Oxford is a whole other topic, and one in which I have little advice to share. In a small department it might be best to look for one in which all the broad areas are covered at least by one enthusiastic and research active lecturer, so that you will be able to identify your favourite subject. A good small department will inevitably have a few specialist groups with at least two academics and maybe some post doctoral research fellows and a few PhD students working in the same area. It is quite usual to move to another university for an MSc or PhD if your specialism is not covered in the department you gain your BSc.

The PhD students are an essential part of university mathematics community. Typically they will teach a few hours a week as "demonstrators", taking problems classes rather than lectures. As they usually not that much older than undergraduates they can often more easily identify with the difficulties of undergraduates, its not so long ago they struggled with the same material. They can also give you their own experience on things like option choices (especially if they did there undergraduate degree at the same department) and perhaps what are the latest exciting areas of mathematics. So choose a department with a vibrant postgraduate community in mathematics. I suppose there is a danger that lecturers and professors with too many PhD students may not have much time to help undergraduates. From my own perspective the ideal balance would be three PhD students per academic (and not all in the same year), but an average of more than three PhD students per academic in the UK would be rare an embarrassment of riches.

Assessing the research reputation of a mathematics department is arguably easier than any objective assessment of the undergraduate experience. First of all there are some obvious indicators, favoured by league table compilers as they are easy to count. Needless to say they do not tell the whole story, but they do give some useful information. The Research Assessment Exercise (RAE) seeks to rate each department on the quality mainly of their "outputs". In practise this is mainly the four best research papers by each academic. The individual or department decides which is the best four over a specified (e.g. 4 year) period, then a panel of experts (other senior mathematicians) decides how good they are. Each department is given a rating, which in the last RAE 2001 was 0 to 5, and 5* (which should probably mean 6). As each department could leave out people by declaring them to be not "research active" (even though they were) this was open to some manipulation and gamesmanship. The letter A-C after the numerical grade indicates what proportion of the staff were included. A means they were all in and C means they were playing the game. Note that the judgement of the quality of the papers is subjective. In contrast to some other disciplines, like for example English Literature, where academics indulge in open warefare over what is "true", mathematicians generally agree on whether a theorem has been proved. Generally you can read the proof of a theorem, and when you understand it agree that it is true or point out a specific flaw. Or you can come up with a counter example. What we argue about is the value of the mathematical result: is it important, or even is the proof elegant. So the judgement is somewhat subjective. Mathematicians in the UK generally agree that Cambridge, Oxford Warwick and Imperial have exceptionally good mathematics departments by anyone's reckoning. But even then mathematicians disagree on which order we should list them. We sometimes say "COWI" as an extension of the idea of "Oxbridge" to the top four in Mathematics. The RAE agrees as Cambridge had three 5*s (Pure, Applied and Stats) while the other three had two 5*s and a 5. But should it be some permutation of those letters? Also Bristol had two 5*s and a 5 (COWIB?). So to some extent the RAE confirms our prejudices, and still leaves the detailed ranking especially of say the top 20 maths departments, as something very much subjective and open to debate. The next RAE is taking place in 2008 so look out for the results of that. There is some indication that the process will be largely abandoned after that as it takes to much time to actually look at all the papers to see how good they are, so it will be replaces by a cruder process. No doubt it will be one that will spark more debate as to its validity.

The research grants held by a department certainly say something about the research activity (I made a list of various League table data including a EPSRC grants held). If there are a lot of grants there is certain to be a lot going on. Also the main government organisation funding research, the EPSRC, has a highly competitive process to get the grants based on reviews by other mathematicians both in the UK and around the world. So if someone has a grant then their work is likely to be highly thought of. That said mathematicians do not necessarily need big grants to do research. The big grants generally pay for postdoctoral research assistants, and especially for highly theoretical research it may be harder to get someone else to "help" prove a theorem than do it your self! Research led maths departments in the UK need the income from grants to maintain their level of activity. In practise the money the governments and the UK (or EU) student contributes is less than it costs the university to educate an undergraduate. This means that your education is subsidised by fees from overseas (non EU) students and by research funding. I admit I oversimplify, and university accounting is extremely complicated. For example it is very hard to say what proportion of the cost of a building, library or member of staff should be attributed to undergraduate or post graduate teaching or research. In any case the fact remains that a decent level of research funding will lead to a department that does not feel so short of resources. You can look up the level of EPSRC grants held for each department on the EPSRC website but these figures should be treated with caution. For example the figures for mathematics do not show grants were say a mathematician collaborates with an engineer or biologist but is not listed first on the grant (principle investigator), also there are other sources of funding including other research councils and industry. Looking up the grants held by one or two of the departments you are looking at might prove quite interesting as it gives an idea of what is new and exciting. The grant descriptions are meant to be read by lay people. You might find it amusing to read these and imagine what value of "lay" the authors of the grant proposal had in mind! Not to mention an interesting topic of conversation at an interview.

Which leads me naturally to a digression on the subject of interviews. Probably I should not confess to such cynicism but as a sixthformer I went to interviews at several departments and found that I if I could distract the interviewer to talk about their favourite mathematical topic it would pleasantly fill up the interview time with no difficult questions or awkward silences, and leave the interviewer with the vague impression I must be very keen as I was so interested in their work. Actually I *was* interested but I am not sure they could reliably ascertain that. Joking aside it is an important rule that for any interview, for a course or job, you do best to be well informed about the department (company) and course (job) and have clear reasons why you chose that one especially. In many cases the result of an interview will be that you will be given the department's standard offer anyway. In some cases giving a good impression will have an important effect if you narrowly miss the specified offer and the admissions tutor may have the discretion to offer you a place anyway. They will be looking for evidence that you will do well on the course and an enthusiasm for mathematics and genuine reasons for your choose of specific course are often good indicators. One highly reputable mathematics department scored students at their interview and then compared the results with the final year marks gained by those students. They found a small negative correlation and so abandoned interviews as a selection mechanism, in favour of a set system of standard offers. Of course this might reflect the interview skills of the academics, but the process is certainly highly subjective.

The interview day is as much an opportunity to find out about the course and the university as it is for them to find out about you. In fact the algorithm that seems to be employed by many prospective students is to look at a few universities chosen largely at random, but including factors such as city/rural location, distance from home etc, and with offers that are with in reach of their predicted grades, go to an interview or visit day, and then choose the one where they felt best because someone was nice to them. A key part of a visit to a university department is your contact with current students. In most cases the students you meet, for example they take you on a tour of campus, will be fairly well disposed towards their chosen course. If they were not reasonably positive they would probably not have volunteered for this job. However i am fairly sure that if there were serious problems with a degree course, for example big arguments over some exams being much harder than others or a significant number of lecturers being incoherent, then you would pick this up from talking to students. What you will more likely be able to pick up will be things that are not obviously good or bad in general but may matter to you. For example how much choice do you have, say if you find your really do not like statistics can you avoid it.

There are a few other indicators of research activity and quality that are relevant and might be easily checked on the department's web site. Many of the most respected senior mathematicians in the UK will have been elected as a "Fellow of the Royal Society" FRS. This strangely named society is what would in most countries be called something like National Academy of Science, but like a lot of things where we were among the first to have such a thing we weren't quite sure what to call it! Election to FRS certainly means that the senior members of the scientific community in the UK respect their contribution, although the Royal Society has itself admitted that there is a bias towards those in London, Oxford and Cambridge. This may be largely an effect of inertia: you are more likely to be elected if the other fellows know about your work, and this is presumably more likely if you live near them. Also Fellow of the Royal Society often seem to live to a ripe old age and they retain their status after they are retired, so even if the potential bias has been eliminated it will take a while to show up in the figures. Of course someone might be a top-notch mathematician and not be an FRS so that should not be held against them! Certainly having a few FRSs (and people of similar standing) among the current (not retired) staff is a good thing, and these are typically people who have made a significant and lasting contribution to mathematics and are well known around the world. Membership of some foreign academies, notably the Russian Academy of Sciences or the US National Academy of Sciences would be considered as having at least the same gravitas. With luck you will be able to take a lecture course taught by them, and perhaps get to know them enough that you can ask them advice about the direction of your mathematical career. There are indications of the presence of the mathematical stars, the top prize in mathematics (equivalent of the Nobel Prize) is the Field's Medal, but fields medallists are somewhat rare, but there are other significant prizes. [2]. One thing to ask current students is if they actually get taught by the super-star mathematicians in their department. It could be that they only teach postgraduate courses.

The strategy of applying to degree courses that you expect just to be able to meet the grades is a common one. It is certainly an understandable one for two reasons. First because it stretches you and gives you the impetus to do the best you can in your exams, and secondly on the basis of "market forces" that the courses asking for higher grades must be the ones people most want to go to, the 'price' going up with the demand. Both these arguments have some merit but let me put an alternative view on both of them. Suppose you expect to just about get three A grades, you are one of the best and keenest maths students at your sixth form college and expect to go far. You apply for a prestigious department and get in. Suddenly you find that not only does everyone there have at least three grade As but they all seem to have done further maths, often as fourth A-level, and some of them took the STEP paper. You have always been near the top of the class at school and suddenly you feel like a small fish in a very big pond. Now a students reaction to this situation varies with their personality. Some people are naturally very competitive and will always try to struggle to get to the top of any pile, others will feel defeated and not perform well in this situation. There is another effect, for which I only have anecdotal evidence, students who are near the top of their class (say the top quartile) often help weaker students by explaining things they have only just learnt themselves. As any teacher knows there is nothing quite like teaching something for making sure you understand it. This also boosts the confidence of the more able student. It is not clear if the result of this is more likely to be a widening gap between the the 'top' and the rest as the more able students gain advantage from their informal teaching, or if their help actually raises the whole level. I would guess that both these effects happen but it would be hard to measure. But the anecdotal evidence suggest many students do best near the top of their class but not way out ahead. Indeed some students tell me that this assumption did actually inform their choice of degree course. Sometimes along with financial incentives for students with three or more A grades to put a course with an AAB or ABB standard offer as their first choice! It is interesting to ponder the fact that courses demanding very high grades (e.g. those at COWI) still produce significant numbers of graduates with lower grades such as lower second and third class degrees. The difficulty of exams, or at least the marking of exams, varies to some extent between institutions but some uniformity is enforced, especially on things like what counts as first, by a system of external examiners. Now for some jobs the degree classification is not so important, a third class degree in engineering does not seem to have held back Carrol Vordeman for example. However a first class degree is generally taken to be an indication of suitability for a place to study for a PhD, and an an upper second often the cut of point for grants for postgraduate study. There are likely to be

[2] The biographies of many of the stars of mathematics, including many dead famous people, can be found on the MacTutor website of St Andrew's University

people who would have gone on to do research in mathematics had they gone to an institution requiring lower grades, it is hard to quantify this but I can think of a few individuals to whom this probably applies. Outside academia some employers especially large companies taking on large numbers of trainees in management or finance, will set thresholds on degree classification along with other selection criteria. Some companies will recruit preferentially from a small number of large universities, based on the premise that they found good people there before and they can't afford to send recruiting teams to too many places. Overall the best strategy is likely to be a compromise between optimising your expected degree classification and the reputation (deserved or not!) of course and university where you study.

Secondly the market forces argument has some flaws as well (this story might be better told by an economist). In a perfect market the consumer is in possesion of all the information they need to make a decision. However potential undergraduates are generally not that well informed. In fact it is almost impossible to know how well suited a degree course will be for you, and you will not even know in retrospect if it was the right choice as you cannot repeat the experiment. To take another economic idea degree course act to some extent as a "luxury good", diamonds for example are highly prized lumps of carbon at least partly as they are expensive. A the demand for a luxury good goes up as the price increases (at least over some interval). So admissions tutors often find that if they dare to make higher offers the number of applicants increases next year. Of course they don't do this dramatically as departments can be heavily penalised for under (actually or over) recruiting. So it is entirely plausible that some courses are not really so good, but students with quite high grades go there. Perhaps the course was really good a few years ago, but some of the lecturers left and the market (with imperfect information) has not yet adjusted. Of course the converse holds: you might get a 'bargain' by going to a department that has recently hired some really good new lecturers but the market has not caught up (or the admissions tutor is too timid to raise the offers). It must also be said that A level grades are not a perfact guide to the enthusiasm and ability of maths students. A-level maths is relatively straightforward to those who love maths, even so simple that it puts some people off and they take a physics degree as it seems harder! It almost has to be that level as otherwise it is much harder for those without a natural ability. On the other hand A-level in chemistry is often baffling for the mathematically inclined seeming like a collection of rules for chemical reactions each rule with so many exception that one wonders if it is a rule at all. I remember hoping that after I had done a mathematics degree I would understand quantum mechanics and Schrodinger equation and then questions of chemistry would have definite mathematical answers. Actually I think the state of the art is that a lot of chemistry can be reduced to computations and computers are now fast enough to make this useful. In any case an ability to bluff you way through A-level chemistry, or indeed more subjective subjects, is not necessarily an indicator of future mathematical ability. UK mathematics departments are increasingly looking for other indicators including STEP papers, or grades on core A-level modules as selection criteria.

What if in the end you don't want to be a professional mathematician? Actually the majority of mathematics graduates do not choose a career that actually uses mathematics at the level to which they have studied. But this certainly does not mean they made a mistake. The skills learned from studying for a mathematics degree including creative problem solving, logical reasoning, critical analysis of an argument, abstraction and generalisation are highly valued across a whole range of possible careers. In contrast to the rest of the world the UK has a culture in which it is permissible anywhere from on a bus to in the national media to admit mathematical incompetence. You only have to say you are studying mathematics or working as a mathematician an people will say “I was never any good at maths”. With no embarrassment! In continental Europe it would be like saying “I never learnt to read”. In anycase for those of us who are natural talented at mathematics despite our national mathematophobic culture this turns out to be an advantage. The scarcity of mathematically skilled people in the UK means that maths (and similarly numerate) graduates command a significantly higher starting salary over non-mathematical graduates with the same degree classification (so our careers service tells me). So you will have plenty of choices after graduating if you decide not to go on to postgraduate study or a career in mathematics. That said an academic career is increasingly attractive. Full bursaries from the ”Doctoral Training Account”, one of the main funding sources that departments have to pay students to do PhDs, cover your fees and currently around 12k per year. But as this is tax free and you get the other financial benefits of being a student so it is closer to many graduate starting jobs in terms of standard of living. So bearing in mind that say by your third year year you might have your sights on career not directly using mathematics one thing to look at is the availability of options courses outside mathematics, for example ones that give you a head start in professional qualifications in for example accountancy or as an actuary. In the case of statistics there is of course a large demand for statisticians in industry, for example in pharmaceutical companies. The entry qualification for a professional statistician this tends to be an MSc, so you would generally choose statistics options and then start looking for an MSc place early in your third year.

Disclaimer: the views expressed in this article are personal opinion of Professor Lionheart at the time of writing and are not intended to reflect any policy of the School of Mathematics or the University of Manchester