Q1. All these are done by applying Bendixson’s test, which states \( \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \) has to change sign for a limit cycle to exist.

1. \[
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 3x^2 + 1 + 3y^2 > 0 \text{ everywhere}
\]

2. \[
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 - (1 + x^2 + x^4) < 0 \text{ everywhere}
\]

3. \[
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 + 1 + x^2 > 0 \text{ everywhere}
\]

Q2. From Bendixion’s test is have

\[
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 2 - y^2 - 1 - x^2 = 1 - x^2 - y^2 = 1 - (x^2 + y^2),
\]

which is clearly always greater than zero inside the unit circle.

Q3. This 2nd order equation is equivalent to the 1st order system

\[
\dot{x} = y, \quad \dot{y} = \epsilon \left( 1 - x^2 - y^2 \right) y - x.
\]

This clearly has an equilibrium point at the origin since when \( x = 0, y = 0 \)
\( \dot{x} = \dot{y} = 0. \)

Now

\[
gr = x\dot{x} + y\dot{y} \Rightarrow \dot{r} = \frac{\epsilon(1 - r^2)y^2}{r}
\]

Hence \( \dot{r} = 0 \) when \( r = 1 \) this is a limit cycle.