Q1.
1. Eigenvalues are $\lambda = 3, 1$ ∴ unstable node
2. Eigenvalues are $\lambda = \frac{5}{2} \pm \frac{i\sqrt{7}}{2}$ ∴ unstable spiral
3. Eigenvalues are $\lambda = \pm 2i$ ∴ centre

Q2.
Q3.

1. \[ \dot{x} = y \quad \dot{y} = \frac{6}{5}y - \frac{7}{5}x \]

Hence the eigenvalues are \( \lambda = -\frac{3}{5} \pm \frac{i\sqrt{26}}{5} \) \( \therefore \) stable spiral.

2. \[ \dot{x} = y \quad \dot{y} = x \]

Hence the eigenvalues are \( \lambda = \pm 1 \) \( \therefore \) this is a saddle.

3. \[ \dot{x} = y \quad \dot{y} = -6x + 5y \]

Hence the eigenvalues are \( \lambda = 2, 3 \) \( \therefore \) a saddle node.

Q4.
Q5.

1. \(4y^2 - x^2 \Rightarrow x = \pm 2\)
   If \(x = -2y\) leads to \(y^2 + y + 2 = 0\) which has no real roots, whereas, \(x = 2y\) gives \(y^2 - y - 2 = 0\), hence \(y = -1\) or \(2\), and leads to two equilibrium points at \((4, 2)\) and \((-2, -1)\).
   Near the equilibrium point \((4, 2)\) the system is approximately
   \[
   \dot{\xi} = 4\xi + 4\eta \quad , \quad \dot{\eta} = -8\xi + 16\eta
   \]
   which has \(\lambda = 10 \pm 2 \geq 0\), hence this is an unstable node.
   Near the equilibrium point \((-2, -1)\) the system is approximately
   \[
   \dot{\xi} = 2\xi - 8\eta \quad , \quad \dot{\eta} = -2\xi + 8\eta
   \]
   which has \(\lambda = -5 \pm i\sqrt{23}\), hence this is a stable spiral.

2. This is equivalent to the 1st order system
   \[
   \dot{x} = y \quad , \quad \dot{y} = -x + x^3
   \]
   Hence the equilibrium points are at \((-1, 0), (0, 0)\) and \((1, 0)\). Near both the first and third equilibrium points the equation is approximately
   \[
   \dot{\xi} = \eta \quad , \quad \dot{\eta} = 2\xi
   \]
   hence \(\lambda = \pm \sqrt{2}\) and these are saddles. Near the origin the equation is approximately
   \[
   \dot{\xi} = \eta \quad , \quad \dot{\eta} = -\xi
   \]
   Hence \(\lambda = \pm i\), and this is a centre.
3. This is equivalent to the 1st order system

\[ \dot{x} = y, \quad \dot{y} = -x + x^3 - y \]

Hence the equilibrium points are at (-1,0), (0,0) and (1,0). Near both the first and third equilibrium points the equation is approximately

\[ \dot{\xi} = \eta, \quad \dot{\eta} = 2\xi - \eta \]

hence \( \lambda = 1, -2 \) and these are saddles. Near the origin the equation is approximately

\[ \dot{\xi} = \eta, \quad \dot{\eta} = -\xi - \eta \]

Hence \( \lambda = -1/2 \pm 1/2i\sqrt{3} \), and this is a stable spiral.

Q6.