How to solve an ODE using DSolve

\[
\text{DSolve}\left[ x'[t] + x[t] - e \cdot (x[t])^2 = 0, x[t], t \right]
\]

\[
\\{(x[t] \rightarrow -\frac{1}{e^{t+C[1]} - e}\}\}
\]

Now with the boundary condition (notice it worries a bit about signs)

\[
\text{In}[1]:= \text{DSolve}\left[\{x'[t] + x[t] - e \cdot (x[t])^2 = 0, x[0] = 1\}, x[t], t\right]
\]

\[
\text{Solve::ifun: Inverse functions are being used by Solve, so some solutions may not be found; use Reduce for complete solution information. :>}
\]

\[
\text{Out}[1]= \\{(x[t] \rightarrow -\frac{1}{e^{t} - e + e^{t} e}\}\}
\]

Let's look at the series for that

\[
\text{In}[9]:= \text{Series}\left[\frac{1}{-e^{t} - e + e^{t} e}, \{e, 0, 1\}\right]
\]

\[
\text{Out}[9]= -e^{t} - e^{-2} t \left(-1 + e^{t}\right) e + O\left[e\right]^2
\]