1. Categorise the following PDEs by order, linearity or degree of nonlinearity, and (if linear) whether homogeneous or inhomogeneous:

   (a) \( u_t - (x^2 + u)u_{xx} = x - t \)
   (b) \( u^2 u_{tt} - \frac{1}{2} u_x^2 + (uu_x)_x = e^u \)
   (c) \( u_t - \nabla^2 u = u^3 \)
   (d) \( (u_{xy})^2 - u_{xx} + u_t = 0 \)
   (e) \( u_t + u_x - u_y = 10 \)

2. Categorise the following 2nd order PDEs as elliptic, parabolic or hyperbolic. Also state their degree of nonlinearity and (if linear) whether homogeneous or inhomogeneous:

   (a) \( u_t + u_{tx} - u_{xx} + u_x^2 = \sin u \)
   (b) \( u_x + u_{xx} + u_y + u_{yy} = \sin(xy) \)
   (c) \( u_x + u_{xx} - u_y - u_{yy} = \cos(xyu) \)
   (d) \( u_{tt} + xu_{xx} + u_t = f(x,t) \)
   (e) \( u_t + uu_{xx} + u^2 u_{tt} - u_{tx} = 0 \)

3. Laplace’s equation for \( u(x,y) \), which is

   \[ u_{xx} + u_{yy} = 0 \]

   with the boundary conditions

   \[ u(x,0) = \gamma \cos(x/\gamma), \quad u_y(x,0) = 0 \]

   has the unique solution

   \[ u(x,y) = \gamma \cosh(y/\gamma) \cos(x/\gamma). \]

   [Conform that this is a solution satisfying the conditions].

   If these solutions were to vary continuously with the boundary conditions we would have:

   For any \((x,y)\) and any \(\delta > 0\), \(\exists \ \epsilon > 0\) such that for all \(|u(x,0)| < \epsilon\) and \(|u_y(x,0)| < \epsilon\), we have \(|u(x,y)| < \delta\).

   Show that this is not the case for the solutions given above for Laplace’s equation.