Week 17

1. A paint company is trying to use up excess quantities of four shades of green paint by mixing them to form a more popular shade. One gallon of the new paint will be made up of $x_1$ gallons of paint 1, $x_2$ gallons of paint 2 etc. Each of the paints is made up of four pigments. If each number represents a percentage, the mixture giving the more popular shade is the solution of the system of equations

\[
\begin{pmatrix}
0 & 80 & 10 & 10 \\
80 & 0 & 30 & 10 \\
16 & 20 & 60 & 72 \\
4 & 0 & 0 & 8
\end{pmatrix}
\begin{pmatrix}
x_1 \\
x_2 \\
x_3 \\
x_4
\end{pmatrix}
=
\begin{pmatrix}
27 \\
40 \\
31 \\
2
\end{pmatrix}.
\]

Find the optimal mixture by solving the system using Gaussian elimination with partial pivoting. (This means that you interchange rows at every step to put the biggest number in modulus on the diagonal. In this example, at the first stage of the elimination you would interchange row one with row two.)

You can check your results in MATLAB (see also the following exercise) by typing the command sequence

```matlab
>> A=[0,80,10;80,0,30;16,20,60,72;4,0,0,8], [L,U,P] = lu(A),
```

2. Given a matrix

\[
A = \begin{pmatrix}
4.00 & -2.00 & 3.00 \\
-2.00 & 7.25 & -2.75 \\
3.00 & -2.75 & 11.50
\end{pmatrix}.
\]

By setting

\[
L = \begin{pmatrix}
1 & 0 & 0 \\
\ell_{21} & 1 & 0 \\
\ell_{31} & \ell_{32} & 1
\end{pmatrix},
\]

and forming the product, find a lower triangular matrix $L$ with ones on the diagonal, and an upper triangular matrix $U$ such that

\[
LU = A.
\]

This is how Gaussian elimination is implemented in MATLAB, the matrix $U$ is the upper triangular matrix formed by GE, the matrix $L$ contains the row mutipliers used at each stage of the elimination, and the matrix $P$ is the permutation matrix needed to effect the partial pivoting. You can check your results by typing the MATLAB command sequence

```matlab
>> A=[4,-2,3;-2,7.25,-2.75;3,-2.75,11.5], [L,U,P] = lu(A),
```