1. The following periodic functions are specified over one period. Sketch them, then expand them in Fourier series. To what limit do you expect the series to converge (use Dirichlet theorem).

(a) \( f(x) = 1 \) for \(-\pi < x < 0\), \( f(x) = 0 \) for \(0 < x < \pi\).
(b) \( f(x) = 0 \) for \(-\pi < x < 0\), \( f(x) = 1 \) for \(0 < x < \pi/2\), \( f(x) = 0 \) for \(\pi/2 < x < \pi\).
(c) \( f(x) = 0 \) for \(-5 < x < 0\), \( f(x) = 3 \) for \(0 < x < 5\).
(d) \( f(x) = x \) for \(0 < x < 1\).
(e) \( f(x) = x^2 \) for \(-1/2 < x < 1/2\).
(f) \( f(t) = |\sin \omega t| \) for \(-\pi < \omega t < \pi\).

2. The following functions are specified over one period. Decide whether they are even or odd, then expand them in Fourier Series.

(a) \( f(x) = -1 \) for \(-\pi < x < 0\), \( f(x) = 1 \) for \(0 < x < \pi\).
(b) \( f(x) = |x| \) for \(-\pi/2 < x < \pi/2\).

3. Given \( f(x) = x \) for \(0 < x < 1\), sketch the even function \( f_e \) of period 2, the odd function \( f_o \) of period 2, and the periodic function \( f_p \) of period 1, each of which equals \( f(x) \) on \(0 < x < 1\). Expand \( f_e \) in a cosine series, \( f_o \) in a sine series, and \( f_p \) in a cosine-sine series.

4. Given \( f(x) = 0\), \(0 < x < 1\); \( f(x) = 1\), \(1 < x < 2\). Sketch the even function \( f_e \) of period 4, the odd function \( f_o \) of period 4, and the periodic function \( f_p \) of period 2, each of which equals \( f(x) \) on \(0 < x < 2\). Expand \( f_e \) in a cosine series, \( f_o \) in a sine series, and \( f_p \) in a cosine-sine series.

5. Use Parseval’s theorem and the result from Problem 2.a to find the sum of the series

\[ 1 + \frac{1}{3^2} + \frac{1}{5^2} + \frac{1}{7^2} + \ldots \]

6. Use Parseval’s theorem and the result from Problem 1.e to find the sum of the series

\[ 1 + \frac{1}{2^4} + \frac{1}{3^4} + \frac{1}{4^4} + \ldots \]

7. Expand the first two functions of problem 1 in Fourier series of complex exponentials, and verify in each case, using Eulers formula, that the answer is equivalent to that one found previously.