1. \( a_0 = 3 \) by inspection (mean \( a_{0/2} = 3/2 \)).

\[ a_n = 0 \text{ for } n \neq 0 \text{ odd} \].

\[ b_n = 2 \int_{0}^{\pi/2} 3 \sin 2\pi n x \, dx = 6 \left[ -\frac{\cos 2\pi n x}{2\pi n} \right]_{0}^{\pi/2} = -\frac{3}{\pi n} \left( \cos (\pi n) - 1 \right) = \frac{3}{\pi n} (1 - (-1)^n) \]

\[ 2 \cdot \frac{3}{\pi n} \text{ for } n \text{ odd} \text{, } 0 \text{ for } n \text{ even} \]

\[ f(x) = \frac{3}{\pi} + \frac{6}{\pi} \sin 2\pi x + \frac{3}{\pi} 6\pi x + \frac{6}{\pi} \sin 10\pi x + \ldots \]

As \( x = 0 \) and \( x = \pi/2 \), series converges to \( 3/2 \), half the jump at the discontinuity and \( 3 = f(\pi/2) \) at \( x = \pi/2 \) where \( f \) is continuous, by Dirichlet's Theorem.

2. \[ C_n = \int_{0}^{\pi/2} 3 \sin 2\pi n x \, dx = \frac{1}{\pi n} (e^{-\pi n} - 1) = \frac{3i}{2 \pi n} (e^{-\pi n} - 1) = \begin{cases} 0 & \text{for } n \text{ even} \\ \frac{3i}{\pi n} & \text{for } n \text{ odd} \end{cases} \]

so \( f(x) = \begin{cases} 0 & \text{for } n \text{ even} \\ \frac{3i}{\pi n} e^{2\pi n x} - \frac{i}{\pi} e^{\pi n x} - \frac{i}{\pi} e^{-\pi n x} + \frac{3i}{\pi} e^{-2\pi n x} + \frac{i}{\pi} e^{-\pi n x} \end{cases} \)

3. a) \[ \left| \begin{array}{ccc}
1 & 1 & 1 \\
2 & 2 & 1 \\
3 & 6 & 2 \\
\end{array} \right| = -2 \left| \begin{array}{cc}
2 & 1 \\
3 & 2 \\
\end{array} \right| = -2 \cdot 3 - 3 = -9 \\
\]

b) \( 2x + 2x \)

c) \( \left( y \cos (z^2 + x) + yz \sin (z^2 + x) \right) \)

4. a) \( 0 \) or \( 0 \)

b) \( \int_{0}^{1} \int_{0}^{1} \sin a x + \sin b y \, dx \, dy = \sin a + b \)

c) \( \phi(x,y) = \sin x + y \), \( \forall \phi = \cos x + j + j = V \).

d) As \( \text{curl} = 0 \), the integral is independent of path by Green's Theorem (or Stokes' Theorem).

Tips: An easy way to work out \( e^{-i\pi n} \) is to notice \( e^{-i\pi} = -1 \) so \( e^{-i\pi n} = (e^{-i\pi})^n \).

- Remember in 4(b) that if \( \text{curl} = 0 \) in a domain with no holes then \( \nabla \cdot \phi = 0 \) for some \( \phi \); in this Q. we work out \( \phi \).

- Remember to state what result you are using eg Dirichlet's Theorem.

If you don't remember the name state the theorem.

- When it asks for "the first so many terms" the order is constant then should \( \cos \) with \( n=1 \), then \( n=2 \) (for the real Fourier series).

- In Q1, it's actually easy to work out the series at \( x = 0 \) and \( \pi/2 \) as only the constant term is non-zero. Sometimes a good check on the series is to work it out for every value of \( \pi \).

\(*\) This was possibly the bit most often missed, although generally the standard was very high.
In (11) (and 12), some people wanted to write something like

$$f(x) = \frac{3}{2} + \sum_{k=1}^{\infty} \frac{6}{\pi k} \sin 2\pi k x \quad \text{but just for } k \text{ odd.}$$

One way to do this is say $k = 2n - 1$ then $n$ goes from 1 to $\infty$, e.g.

$$\frac{3}{2} + \sum_{n=1}^{\infty} \frac{6}{\pi (2n-1)} \sin 2\pi (2n-1) x$$

Or you can say

$$\frac{3}{2} + \sum_{k=1}^{\infty} \frac{6 \sin 2\pi k x}{\pi k} \quad \text{& k odd}$$

If you like.