2Q2 Part 2 Example Sheet 1 Answers: warm up questions - determinants and eigenvalues

1. (a) 3, (b) -3, (c) 12, (d) 0, (e) 0, (f) -1, (g) 0.

2. (a) \((1 - \lambda)(3 - \lambda) = 0\), \(\lambda = 1, 3\)
   (b) \((2 - \lambda)(4 - \lambda) - 4 = \lambda^2 - 6\lambda + 8 = 0\), \(\lambda = 3 \pm \sqrt{5}\)
   (c) \(\lambda^2 - 7\lambda = 0\), \(\lambda = 0, 7\)
   (d) \(\lambda^2 - 2\lambda = 0\), \(\lambda = 0, 2\)
   (e) \(\lambda^2 - \lambda - 1 = 0\), \(\lambda = 1 \pm \sqrt{5}/2\)
   (f) \(-\lambda^3 + 6\lambda^2 - 11\lambda + 6 = 0\), \(\lambda = 1, 2, 3\)

3. For example (any non-zero scalar multiple will do)
   (a) \((1 - 1)^T\)
   (b) \((2 1)^T\)
   (c) \((3 - 1)^T\)
   (d) \((1 -(\sqrt{5} + 1)/2)^T\)

4. (a) \(\lambda = 8, 2\), eigenvectors \((1 1)^T, (1 - 1)^T\)
   (b) \(\lambda = 5 \pm \sqrt{13}\), eigenvectors \((\frac{2}{4} + \frac{1}{4} (-4 \pm \sqrt{13})\ 1)^T\). Note although this looks complicated you know that for a rank one matrix like the columns must be multiples of each other and that makes it simpler.
   (c) \(\lambda = 1 \pm \sqrt{2}\), eigenvectors \((2 \pm \sqrt{2}, 1)^T\)
   (d) This one is complex but you just do it in exactly the same way. \(\lambda = \frac{1}{2} (9 + i\sqrt{77})\), eigenvectors \((1 + \frac{i}{8} (-9 - i\sqrt{77}), 1))^T\)