1. Masses $m_1$ and $m_2$ interact with no external forces so that their velocities $v_1$ and $v_2$ and their momenta $p_1$ and $p_2$ change. True or false?

(a) $p_2 = m_1 v_2$

(b) $p_1 - p_2 = 0$

(c) $\frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2$ is constant

(d) $p_1 - p_2 = 2m_2 v_2$

(e) $\frac{d}{dt} (p_1 \cdot v_1 + p_2 \cdot v_2) = 0$ if energy is conserved.

(f) $p_1 + m_2 v_2$ is constant

2. (a) A rocket of mass $m$ ejects mass at a speed $S$ relative to the rocket. If $v$ is the speed of the rocket measured in the direction opposite to that in which the mass is ejected and no external forces are applied show that

$$\frac{dv}{dt} + S \frac{dm}{dt} = 0$$

If $S$ is constant and $v = v_0, m = m_0$ when $t = 0$ show that

$$v = v_0 + S \ln \frac{m_0}{m}$$

(b) If $S$ is a function of $m$ and $v$ and $v = v_0, m = m_0$ when $t = 0$ find $v$ as a function of $m$ in the following cases

i. $S = S_0 \frac{m}{m_0}$

ii. $S = S_0 \frac{m_0}{m}$

iii. $S = Av$

iv. $S = B \frac{2m^2}{S_0^2}$

where $S_0, m_0, A$ and $B$ are constants

(c) If $S$ is constant and $v = v_0, m = m_0$ and $r = 0$ when $t = 0$ find $v$ and $r$ as functions of time $t$ in cases for which the rate of mass decrease $\frac{dm}{dt}$ is given by

i. $\frac{dm}{dt} = -B$ ( $B > 0$ constant)

ii. $\frac{dm}{dt} = -\alpha m$ ( $\alpha > 0$ constant)

iii. $\frac{dm}{dt} = -\beta m^2$ ( $\beta > 0$ constant)

3. Two stars made of solid rubber have a force of attraction between them of $F = kx$ where $x$ is the distance between their surfaces. The stars have an equal mass $m$ and at some time $t = 0$ they collide with relative speed $S_0$. All motion stays in one straight line and the total momentum is zero. If $r_e$ is the coefficient of restitution then
(a) What is the relative speed just after the n-th collision?
(b) What is the kinetic energy of each star just after the n-the collision?
(c) What is the time between the n-th collision and the next collision?
(d) How far apart do the stars move after the n-th collision?
(e) Are all collision completed after some finite time? If so what is the time?

4. Sketch the return maps for each of parts (a) to (d) of Question 3.