For this year (academic year 2001/2002) you can expect a very similar paper to those given over the last 3 years. Referring to the year in which the course was taught, you have the 1998 paper as a handout and the 1999 and 2000 papers are on my web page. All questions on those papers are on topics covered this year. This year I did coefficient of restitution and the bouncing ball more thoroughly than last year so I am providing suitable practise questions. Also you should understand the phase plots of simple harmonic motion and the circular pendulum. Unlike last year, but like previous years, all questions will be equally weighted and you will have a choice of 3 from 4. Do not rely on questions covering only one topic. For example there could be a dimensional analysis question combined with population models or Kepler’s laws. Be sure to practise the different types of rocket questions, see problem sheet 7, and exam 1999.

1. (a) Representing base dimensions by \([\text{mass}] = M, [\text{length}] = L\) and \([\text{time}] = T\), which of the following quantities are assigned their correct dimensions and which are not? Give the correct dimensions in cases that are wrong.

\[
\begin{align*}
\text{[velocity]} &= LT \\
\text{[density]} &= MT \\
\text{[acceleration]} &= LT^{-2} \\
\text{[force]} &= MLT^{-2} \\
\text{[impulse]} &= MLT^{-1} \\
\text{[momentum]} &= MLT \\
\text{[angle]} &= T \\
\text{[area]} &= L^2
\end{align*}
\]

(b) Two objects with the same mass collide, moving in one direction only with no external forces. Initially the second is at rest. If the first mass is at rest after the collision find the velocity of the second. Is kinetic energy conserved?

(c) The coefficient of restitution of two colliding objects is defined as

\[
R = \frac{\text{relative speed of moving apart}}{\text{relative speed of moving together}}.
\]

i. Taking all movements to be along a line a particle of mass \(m_1\) moving at velocity \(s\) hits a particle of mass \(m_2\) at rest. After the collision the velocities of the particles are \(v_1\) and \(v_2\).

ii. Write down an equation representing conservation of momentum, and give a formula for the coefficient of restitution \(R\) in terms of \(v_1, v_2\) and \(s\). Hence show that

\[
v_1 = \frac{m_1 - R m_2}{m_1 + m_2} s
\]

and find a similar equation of \(v_2\).

iii. Show that the ratio of kinetic energy before the collision to after the collision is given by

\[
\frac{\frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2}{\frac{1}{2} m_1 s^2} = \frac{m_1^2 + (1 + R^2) m_1 m_2 + R^2 m_2^2}{m_1^2 + 2 m_1 m_2 + m_2^2}
\]

Explain why \(0 \leq R \leq 1\) and give a physical interpretation of the extreme cases \(R = 0\) and \(R = 1\)
2. (a) Representing the “base dimensions” by

\[
\begin{align*}
[\text{mass}] &= M \\
[\text{length}] &= L \\
[\text{time}] &= T \\
[\text{temperature}] &= \Theta
\end{align*}
\]

Which of the following quantities are correctly expressed in terms of the base quantities and which are not? If not give the correct dimensions.

- [speed] = \(TL^{-1}\)
- [area] = \(L^{-2}\)
- [velocity] = \(LT\)
- [density] = \(ML^3\)
- [acceleration] = \(MLT^{-1}\)
- [force] = \(ML^2T^{-2}\)
- [momentum] = \(MTL^{-1}\)
- [angle] = 1
- [energy] = \(\Theta^2L^{-3}\)
- [power] = \(TL^2M\)

(b) A particle of mass \(m\) is projected from the point \(r = (0, 0)\) at time \(t = 0\) with initial velocity \(v_0\).

Assuming that the only force acting on the particle during its flight is the uniform gravitational force \((0, -mg)\)

i. Find formulas for the position and velocity of the particle as functions of time.

ii. Find the distance travelled before the particle returns to the initial height, and the maximum height reached. At what times are these heights attained?

iii. Find and expression for the total energy of the particle, and indicate when the potential and kinetic energy are at their maximum values.

(c) The coefficient of restitution of two colliding objects is defined as

\[
\mathcal{R} = \frac{\text{relative speed of moving apart}}{\text{relative speed of moving together}}.
\]

A ball bounces vertically on a hard, flat, horizontal surface.

i. Assuming there is no air resistance use conservation of energy to show that the ball falls back to the surface with the same speed that it left.

ii. If \(s_n\) is the speed of the ball before it hits the surface on the \(n\) th bounce show that \(s_n = \mathcal{R}^{n-1}s_1\)

iii. Show that during the flight between the \(n\) th and \(n+1\) st bounce the velocity is \(v = s_n - gt\) and height \(y = s_n t - \frac{1}{2}gt^2\) where \(t\) is the time since the last bounce.

Find the maximum height \(y_n\) and time \(t_n\) between bounces.

iv. Show that \(t_n = \mathcal{R}t_{n-1}\).

And hence show that for \(\mathcal{R}\) all the bounces take place during a finite interval of time. If the time taken for the first bounce is 1.0s and all the bounces are completed in 3.0s, find the coefficient of restitution.