1 Introduction

Electrical Impedance Tomography (EIT) is the recovery of the conductivity (or conductivity and permittivity) of the interior of a body from a knowledge of currents and voltages applied to its surface. In geophysics, where the method is used in prospecting and archaeology, it is known as electrical resistivity tomography. In industrial process tomography it is known as electrical resistance tomography or electrical capacitance tomography. In medical imaging, when at the time of writing it is still an experimental technique rather than routine clinical practice, it is called EIT. A very similar technique is used by weakly electric fish to navigate and locate prey and in this context it is called electrosensing.

The simplest mathematical formulation of inverse problem of EIT can be stated as follows. Let \( \Omega \) be a conducting body described by a bounded domain in \( \mathbb{R}^n, n \geq 2 \), with electrical conductivity a bounded and positive function \( \gamma(x) \) (later we will consider also \( \gamma \) complex). In absence of internal sources, the electrostatic potential \( u \) in \( \Omega \), is governed by the elliptic partial differential equation

\[
L_{\gamma} u := \nabla \cdot (\gamma \nabla u) = 0 \quad \text{in} \quad \Omega. \tag{1.1}
\]

It is natural to consider the weak formulation of 1.1 in which \( u \in H^1(\Omega) \) is a weak solution to (1.1). Given a potential \( \phi \in H^{1/2}(\partial \Omega) \) on the boundary, the induced potential \( u \in H^1(\Omega) \) solves the Dirichlet problem

\[
\begin{cases}
L_{\gamma} u = 0 \quad \text{in} \quad \Omega, \\
u|_{\partial \Omega} = \phi
\end{cases} \tag{1.2}
\]

The currents and voltages measurements taken on the surface of \( \Omega, \partial \Omega \), are given by the so-called Dirichlet-to-Neumann map (associated with \( \gamma \)) or voltage-to-current map
Here, \( \nu \) denotes the unit outer normal to \( \partial \Omega \) and the restriction to the boundary is considered in the sense of the trace theorem on Sobolev spaces. We require that \( \partial \Omega \) be at least Lipschitz continuous and \( \gamma \in L^\infty(\Omega) \) with \( \text{ess inf} \text{Re} \gamma = m > 0 \).

The **forward problem** under consideration is the map \( \gamma \in \mathcal{D}_m \mapsto \Lambda_\gamma \), where \( \mathcal{D}_m = \{ \gamma \in L^\infty(\Omega) | \text{ess inf} \gamma \geq m \} \). The **inverse problem** for complete data is then the recovery of \( \gamma \) from \( \Lambda_\gamma \). As is usual in inverse problems we will consider the questions of (1) uniqueness of solution (or from a practical point of view sufficiency of data) (2) stability/instability with respect to errors in the data and (3) practical algorithms for reconstruction. It is also worth pointing out to the reader who is not very familiar with EIT the well known fact that the behavior of materials under the influence of external electric fields is determined not only by the electrical conductivity \( \gamma \) but also by the electric permittivity \( \varepsilon \), so that the determination of the complex valued function \( \gamma(x,\omega) = \sigma(x) + i\omega\varepsilon(x) \), would be the more general and realistic problem, where \( i = \sqrt{-1} \) and \( \omega \) is the frequency. The simple case where \( \omega = 0 \) will be treated in this work. For a description of the formulation of the inverse problem for the complex case we refer for example to [17]. Before we address questions (1)-(3) mentioned above, we will consider how the problem arises in practice.

### 1.1 Measurement systems and physical derivation

For the case of direct current, that is the voltage applied is independent of time, the derivation is simple. Of course here \( \Omega \subset \mathbb{R}^3 \). Let us first suppose that we can apply an arbitrary voltage \( \phi \in H^{1/2}(\Omega) \) to the surface. We assume that the exterior \( \mathbb{R}^3 \setminus \Omega \) is an electrical insulator. An electric potential (voltage) \( u \) results in the interior and the current \( J \) that flows satisfies the continuum Ohm’s law \( J = -\gamma \nabla u \) the absence of current sources in the interior is expressed by the continuum version of Kirchoff’s law \( \nabla \cdot J = 0 \) which together result in 1.1. The boundary conditions are controlled or measured using a system of conducting electrodes which are typically applied to the surface of the object. In some applications, especially geophysical, these may be spikes that penetrate the object, but it is common to model these as points on the surface. Systems are used that to a reasonable approximation apply a known current on (possibly a subset) or electrodes and measure the voltage that results on electrodes (again possibly a subset, in some cases disjoint from those carrying a non-zero current). In other cases it is a predetermined voltage applied to electrodes and the current measured; there being practical reasons determined by electronics or safety for choosing one over the other. In medical EIT applying known currents and measuring voltages is typical. One reason for this is the desire to limit the maximum current for safety reasons. In practice the circuit that delivers a predetermined current can only do so while the voltage required to do that is within a certain range so both maximum current and voltage are limited. For
an electrode (let us say indexed by \( \ell \)) not modelled by a point but covering a region \( E_\ell \subset \partial \Omega \) the current to that electrode is the integral

\[
I_\ell = \int_{E_\ell} -J \cdot \nu \, dx.
\]

(A.3)

Away from electrodes we have

\[
\gamma \frac{\partial u}{\partial \nu} = 0, \quad \text{on } \partial \Omega \setminus \bigcup_{\ell=1}^{L} E_\ell
\]

(A.4)

as the air surrounding the object is an insulator. On the conducting electrode we have \( u|_{E_\ell} = V_\ell \) a constant, or as a differential condition

\[
\nu \times \nabla u = 0 \quad \text{on } \partial \Omega \setminus \bigcup_{\ell=1}^{L} E_\ell
\]

(A.5)

Taken together (A.3)-(A.5) are called the shunt model. This ideal of a perfectly conducting electrode is of course only an approximation, and we note that while the condition \( u \in H^1(\Omega) \) is a sensible condition, ensuring finite total dissipated power, it is not sufficient to ensure 1.3 is well defined. Indeed for smooth \( \gamma \) and smooth \( \partial E_\ell \) the condition results in a square root singularity in the current density on the electrode. We will come back to a more realistic model of electrodes.

It is more common to use alternating current in geophysical and process monitoring applications, and essential in medical applications. Specifically the direction of the current must be reversed within a sufficiently short time to avoid electrochemical effects. This also means that the time average of the applied current should be zero. In medical applications current in one direction for sufficient duration would result in transport of ions, and one of the effects of this can be stimulation of nerves. It would also degrade electrode behavior due to charge build up and ionic changes in the electrode. As a general rule higher levels of current and voltage are considered safer at higher temporal frequencies. The simplest EIT system therefore operates at a fixed frequency using an oscillator or digital signal processing to produce a sinusoidal current. Measurements are then taken of the magnitude, or in the some cases the components that are in phase and \( \pi/2 \) out of phase with the original sine wave. Of course when current or voltage is first applied to the object a transient results, and typical EIT systems are designed to start measuring after this transient term has decayed so as to be negligible.

In geophysics a technique that is complementary to ERT called induced polarization tomography IPT is used to find polarizable minerals. In effect this uses a square wave pulse and measures the transient response [77]. In process tomography a technique known as electrical capacitance tomography is designed for imaging insulating materials with different dielectric permittivities, for example oil and gas in a pipe[50] [93]. Again square waves or pulses are used.
In medical and geophysical problems the response of the materials may vary with frequency. For example in a biological cell higher frequency current might penetrate a largely capacitive membrane and so be influence by the internal structures of the cell while lower frequency currents pass around the cell. This has led to Electrical Impedance Tomography Spectroscopy (EITS)[45], and in geophysics Spectral Induced Polarization Tomography (SIPT)[77]. The spectral response can be established either by using multiple sinusoidal frequencies or by sampling the transient response to a pulse.

Our starting point for the case of alternating current is the time harmonic Maxwell equations at a fixed angular frequency $\omega$. Here it is assumed that the transient components of all fields are negligible and represent the time harmonic electric and magnetics vector fields using the complex representation $F(x,t) = \text{Re}(F \exp(i\omega t))$ and we have

$$\nabla \times \mathbf{E} = -i\omega \mathbf{B}$$

$$\nabla \times \mathbf{H} = \mathbf{J} + i\omega \mathbf{D}$$

The electric and magnetic fields $\mathbf{E}$ and $\mathbf{H}$ are are related to the current density $\mathbf{J}$ electric displacement $\mathbf{D}$ and magnetic flux $\mathbf{B}$ by the material properties conductivity $\sigma$, permittivity $\epsilon$ and permeability $\mu$ by

$$\mathbf{J} = \sigma \mathbf{E}, \quad \mathbf{D} = \epsilon \mathbf{E}, \quad \mathbf{B} = \mu \mathbf{H}$$

The fields $\mathbf{E}$ and $\mathbf{H}$ evaluated on directed curves, while the “fluxes” $\mathbf{J}, \mathbf{D}$ and $\mathbf{B}$ on surfaces. generally cochains. In biomedical applications one can typically take $\mu$ to be constant and to be the same inside the body as outside in air. In non-destructive testing and geophysical applications there may well be materials with differing permeability. We are also assuming linear relations in 1.8. For example the first is the continuum Ohm’s law. We allow for the possibility that the material properties are frequency dependent. In this, dispersion is important in EIS and SIPT. For the moment we also assume isotropy (so that the material properties are scalars).

There are many inverse problems governed by time harmonic Maxwell’s equations. For very large values of $\omega$ this includes optical and microwave tomographic techniques and scattering problems such as radar which we do not discus in this chapter. There are also systems where the fields arise from alternating current in a coil, and measurements are made either with electrodes or with other coils. Mutual (or magnetic) induction tomography (MIT) falls in to this category has been tried in medical and process monitoring applications [46]. In these cases the eddy current approximation [9] approximation to Maxwell’s equations is used. While for direct current EIT (that is ERT) the object is assumed surrounded by an insulator, in MIT one must account for the magnetic fields in the surrounding space, there being no magnetic ‘shielding’.

We now come to the assumptions used to justify the usual mathematical model of EIT that are distinct from many other inverse problems for Maxwell’s equations. We already have

$$\nabla \times \mathbf{E} = -i\omega \mathbf{B}$$

$$\nabla \times \mathbf{H} = \mathbf{J} + i\omega \mathbf{D}$$

$$\mathbf{J} = \sigma \mathbf{E}, \quad \mathbf{D} = \epsilon \mathbf{E}, \quad \mathbf{B} = \mu \mathbf{H}$$
Assumption 1 Transients components of all fields are negligible.

which simply means we have waited a sufficient ‘settling time’ before making measurements.

We are interested in relatively low frequencies where magnetic effects can be neglected this translates in to two assumptions

Assumption 2 $\omega\sqrt{\epsilon\mu}$ is small compared with the size of $\Omega$

This means that the wavelength of propagating waves in the material is large. A measurement accuracy of $2^{-12} = 1/4096$ is ambitious at higher frequencies means that for wave effects to be negligible

$$d\frac{\omega}{\sqrt{\epsilon\mu}} < \cos^{-1}\frac{4095}{4096}$$

(1.9)

where $d$ is the diameter of the body. Taking the relative permittivity to be 10 and $R = 0.3m$ gives a maximum frequency of 1MHz

Assumption 3 $\sqrt{\omega\sigma\mu/2}$ is small compared with the size of $\Omega$

The quantity

$$\delta = \sqrt{\frac{2}{\omega\sigma\mu}}$$

(1.10)

is known as the skin depth. For a frequency of 10kHz and a conductivity of $0.5$Sm$^{-1}$ typical in medical applications, the skin depth is 7m. In geophysics lower frequencies are typical but length scales are larger. In a conducting cylinder the electric field decays with distance $r$ from the boundary at a rate $e^{-r/\delta}$ due to the opposing magnetic field. At EIT frequencies this simple example suggests that accurate forward modelling of EIT should take account of this effect although it is currently not thought to be a dominant source of error.

The effect of assumptions 2 and 3 combined is that we can neglect $\nabla \times E$ in Maxwell’s equations resulting in the standard equation for complex EIT

$$\nabla \cdot (\sigma + i\omega\epsilon) \nabla u = 0$$

(1.11)

here the expression $\gamma = \sigma + i\omega\epsilon$ is called complex conductivity, or logically the admittivity. While $1/\sigma$ is called resistivity and the rarely-used complex $1/|\gamma|$ impedance. A scaling argument is given for the approximation (1.11)in [30], and numerical checks on the validity of the approximation in [36] and [96].

It is often not so explicitly stated but while in the direct current case one can neglect the conductivity of the air surrounding the body, for the alternating current case the electrodes are coupled capacitively, and while $\sigma$ can be assumed to be zero for air the permittivity of any material is no smaller than that of a vacuum $\epsilon_0 = 8.85 \times 10^{-12}$ although dry air approaches that value. One requires then

Assumption 4 $\omega\epsilon$ in the exterior is negligible compared to $|\sigma + i\omega\epsilon|$ in the interior.
Figure 1: A system of electrodes used for chest EIT at Oxford Brookes University. The positions of the electrodes was measured manually with a tape measure and the cross sectional shape was also determined by manual measurements. These electrodes have a disk of jell containing silver chloride solution that makes contact with the skin. Each electrode was attached to the EIT system by a screened lead, not shown in this picture for clarity.

For example with a conductivity or $0.2 \, S \, m^{-1}$ the magnitude of the exterior admittivity reaches $2^{-12}$ of that value for a frequency of 0.88 MHz. For a more detailed calculation the capacitance between the electrodes externally could be compared with the impedance between electrodes. In ECT frequencies above 1 MHz are used and the exterior capacitance can not be neglected. Indeed an exterior grounded shield is used so that the exterior capacitive coupling is not affected by the surroundings (see fig 2).

1.2 The concentric anomaly: a simple example

A simple example helps us to understand the instability in the inverse conductivity problem. Let $\Omega$ be the unit disk in $\mathbb{R}^2$ with polar coordinates $(r, \theta)$ and consider a concentric anomaly in the conductivity of radius $\rho < 1$

$$\gamma(x) = \begin{cases} a_1 & |x| \leq \rho \\ a_0 & \rho < |x| \leq 1 \end{cases}$$

(1.12)

From separation of variables, matching Dirichlet and Neumann boundary conditions at $|x| = \rho$ we find for $n \in \mathbb{Z}$

$$\Lambda_\gamma e^{in\theta} = |n| \frac{1 + \mu |n|^2}{1 - \mu |n|^2} e^{in\theta}$$

(1.13)

where $\mu = (a_1 - a_0)/(a_1 + a_0)$. From this one sees the effect of the Dirichlet to Neumann map on the complex Fourier series, and the effect on the real Fourier series is easily deduced. This example was considered in [58] as an example of the eigenvalues and eigenfunctions of $\Lambda_\gamma$, and also by [2] as an example of instability. We see that $||\gamma - a_0||_{L^\infty(\Omega)} = |a_1 - a_0|$ independently of $\rho$ and yet $\Lambda_\gamma \to \Lambda_{a_0}$ in the operator norm. Hence if an inverse map $\Lambda_\gamma \mapsto \gamma$ exists it cannot be continuous in this topology. Similar arguments can be used to show instability of inversion in other norms.
This example reveals many other features of the more general problem. For example experimentally one observes saturation: for an object placed away from the boundary changes in the conductivity of an object with a conductivity close to the background are fairly easily detected but but for an object of very high or low conductivity further changes in conductivity of that object have little effect. This saturation effect was explored for offset circular objects (using conformal mappings) by Seagar[90]. This is also an illustration of the non-linearity of $\gamma \rightarrow \Lambda_\gamma$. One can also see in this example that smaller objects (with the same conductivity) produce smaller changes in measured data as one might expect.

On the unit circle $S^1$ one can define an equivalent norm on the Sobolev space $H^s(S^1)$ (see definitions in Section ??) by

$$|| \sum_{n=-\infty, n \neq 0}^{\infty} c_n m r e^{i n \theta} ||^2_s = \sum_{n=-\infty, n \neq 0}^{\infty} n^{2s} c_n^2$$

It is clear for this example $\Lambda_\gamma : H^s_0(S^1) \rightarrow H^{s-1}_0(S^1)$ for any $s$. Roughly the current is a derivative of potential and one degree of differentiability less smooth. Technically $\Lambda_\gamma$ (for any positive $\gamma \in C^\infty(\Omega)$) is a first order pseudo-differential operator [73]. The observation that for our example $e^{-i n \theta} \Lambda_\gamma e^{i n \theta} = |n| + o(n^{-p})$ as $|n| \rightarrow \infty$ for any $p > -1$ illustrates that the change in conductivity and radius of the interior object is of somewhat secondary importance! In the language of pseudodifferential operators for a general $\gamma$ such that $\gamma - 1$ vanishes in a neighbourhood of the boundary, $\Lambda_\gamma$ and $\Lambda_1$ differ by a smoothing operator.

We see also from 1.13 that $\Lambda_\gamma^{-1}$ is also well defined operator on $L^2_0 \rightarrow L^2$ with eigenvalues $O(|n|^{-1})$ and is therefore a Hilbert-Schmidt operator. This is also known for the general case[35].

Early work on medical applications of EIT[53],[66] hoped that the forward
problem in EIT would be approximated by generalized ray transform – that is integrals along current stream lines. The example of a concentric anomaly was used to illustrate that EIT is nonlocal [91]. If one applies the voltage \( \cos(\theta + \alpha) \), which for a homogeneous disk would result in current streamlines that are straight and parallel, a change in conductivity in a small radius \( \rho \) from the centre changes all measured currents. Not just on lines passing through the region of changed conductivity \(|x| \leq \rho\). In the 1980s a two dimensional algorithm that backprojected filtered data along equipotential lines was popularized by Barber and Brown [12]. Berenstein [15] later showed that the linearized EIT problem in a unit disc can be interpreted as the Radon transform with respect to the Poincaré metric and a convolution operator and Barber and Brown’s algorithm is an approximate inverse to this.

In process applications of EIT and related techniques the term soft field imaging is used, which by analogy to soft field X-rays means a problem that is non-linear and non-local. However in the literature when the ‘soft field effect’ invoked it is often not clear if it is the nonlinear or non local aspect to which they refer and in our opinion the term is best avoided.

1.3 Measurements with electrodes

A typical electrical imaging system uses a system of conducting electrodes attached to the surface of the body under investigation. One can apply current or voltage to these electrodes and measure voltage or current respectively. For one particular measurement the voltages (with respect to some arbitrary reference) are \( V_\ell \) and the currents \( I_\ell \), which we arrange in vectors as \( V \in \mathbb{C}^L \) and \( I \in \mathbb{C}^L \). The discrete equivalent of the Dirichlet-to-Neumann map is the transfer admittance, or mutual admittance, matrix \( Y \) which is defined by \( I = YV \).

It is easy to see that the vector \( 1 = (1, 1, \ldots, 1)^T \) is in the null space of \( Y \), and that the range of \( Y \) is orthogonal to the same vector. Let \( S \) be the subspace of \( \mathbb{C}^L \) perpendicular to \( 1 \) then it can be shown that \( Y|_S \) is invertible from \( S \) to \( S \). The generalized inverse (see chapter 28 ?) \( Z = Y^+ \) is called the transfer impedance. This follows from uniqueness of solution of shunt model boundary value problem.

The transfer admittance, or equivalently transfer impedance, represents a complete set of data which can be collected from the \( L \) electrodes at a single frequency for a stationary linear medium. It can be seen from the weak formulation of (1.11) that \( Y \) and \( Z \) are symmetric (but for \( \omega \neq 0 \) not Hermitian). In electrical engineering this observation is called reciprocity. The dimension of the space of possible transfer admittance matrices is clearly no bigger than \( L(L - 1)/2 \), and so it is unrealistic to expect to recover more unknown parameters than this. In the analogous case of planar resistor networks with \( L \) ‘boundary’ electrodes the possible transfer admittance matrices can be characterized completely [32], a characterization which is known at least partly to hold in the planar continuum case [57]. A typical electrical imaging system applies current or voltage patterns which form a basis of the space \( S \), and measures some subset of the resulting voltages which as they are only defined up to an
additive constant can be taken to be in $S$.

We have seen that the shunt model is non-physical. In medical application with electrodes applied to skin, and in “phantom” tanks used to test EIT systems with ionic solutions in contact with metal electrodes a contact impedance layer exists between the solution or skin and the electrode. This modifies the shunting effect so that the voltage under the electrode is no longer constant. The voltage on the electrode is still a constant $V_\ell$ so now on $E_\ell$ there is a voltage drop across the contact impedance layer

$$\phi + z_\ell \sigma \frac{\partial \phi}{\partial \nu} = V_\ell$$

where the contact impedance $z_\ell$ could vary over $E_\ell$ but is usually assumed constant. Experimental studies have shown [52] that a contact impedance on each electrode is required for an accurate forward model. This new boundary condition together with 1.3 and 1.4 form the Complete Electrode Model or CEM. For experimental validation of this model see [29], theory [95] and numerical calculations [87, 104]. A nonzero contact impedance removes the singularity in the current density, although high current densities still occur at the edges of electrodes (fig 3). For further details on the singularity in the current density see [34].

The set of imposed current patterns, or excitation patterns is designed to span $S$, or at least that part of it that can be accurately measured in a given situation. In medical EIT, with process ERT following suit, early systems designed at Sheffield [12] assumed a two dimensional circular domain. Identical electrodes were equally spaced on the circumference, and taking them to be numbered anticlockwise the excitation patterns used were adjacent pairs, that is proportional to

$$I_i^\ell = \begin{cases} 1 & i = \ell \\ -1 & i = \ell + 1 \\ 0 & \text{otherwise} \end{cases}$$

for $i = 1, \ldots, L - 1$. The electronics behind this is balanced current source connected between two electrodes[54, Ch ?], and this is somewhat easier to achieve in practice than a variable current source at more than two electrodes. For general geometries where the electrodes are not placed on a closed curve other pairs of electrodes are chosen. For example $I_1^1 = -1$ while $I_i^1 = \delta_{i\ell}, \ell \neq 1$

Measurements of voltage can only be differential and so voltage measurements are taken between pairs of electrodes, for example adjacent pairs, or between each and some fixed electrode. In pair drive systems similar to the original Sheffield system voltages on electrodes with nonzero currents are not measured, resulting in incomplete knowledge of $Z$.

In geophysical surface resistivity surveys it is common to use a pair drive and pair measurement system, using electrodes in a line where a two dimensional approximation is used, or laid out in a rectangular or triangular grid where the full three dimensional problem is solved. Measurements taken between pairs of non-current carrying electrodes. The choice of measurement strategy is limited
(a) Current density on the boundary for passive and active electrodes. In fact there is a jump discontinuity at the edge of electrodes for non-zero contact impedance although our plotting routine has joined the left and right limits.

(b) The effect of contact impedance on the potential beneath an electrode. The potential is continuous.

(c) Interior current flux near an active electrode

(d) Interior current flux near a passive electrode

Figure 3: The current density on the boundary with the CEM is greatest at the edge of the electrodes, even for passive electrodes. This effect is reduced as the contact impedance increases. Diagrams courtesy of Andrea Borsic
by the physical work involved in laying out the cables and by the switching sys-
tems. Often electrodes will be distributed along one line and a two dimensional
approximate reconstruction used as this gives adequate information for less cost.
A wider spacing of the current electrodes is used where the features of interest
at a greater depth below the ground. In another geophysical configuration, cross
borehole tomography, electrodes are deployed down several vertical cylindrical
holes in the ground, typically filled with water, and current passed between
electrodes in the same or between different bore holes. Surface electrodes may
be used together with those in the bore holes. In some systems the current is
measured to account for a non-ideal current source.

In capacitance tomography a basis of voltage patterns is applied and the
choice $V_i^j = \delta_{ij}$ is almost universal. The projection of these vectors to $S$ (we call
an “electrode-wise basis”) is convenient computationally as a current pattern.

Given a multiple drive system capable of driving an arbitrary vector of cur-ents in $S$ (in practice with in some limits on the maximum absolute current and
on the maximum voltage) we have a choice of excitation patterns. While exact
measurements of $Z_\mathbf{I}$ for $\mathbf{I}$ in any basis for $S$ is clearly sufficient the situation is
more complicated with measurements of finite precision in the presence of noise.
If a redundant set of currents is taken the problem of estimating $\mathbf{Z}$ becomes one
of multivariate linear regression. The choice of current patterns is then a design
matrix. Another approach seeks the minimum set of current patterns that re-
results in usable measurements. Applying each current pattern and taking a set of
measurements takes a finite time, during with the admittivity changes. Without
more sophisticated statistical methods (such as Kalman filters [105]) there are
diminishing returns in applying redundant current patterns. Suppose that the
total power $\mathbf{V}^\mathbf{T}Z\mathbf{I}$ is constrained (we want to keep our patient electrically safe)
and the current best estimate of the admittivity is gives a transfer admittance
$Z_{\text{calc}}$, then it is reasonable to apply currents $\mathbf{I}$ such that $(Z - Z_{\text{calc}})\mathbf{I}$
is above the threshold of voltages that can be accurately measured and modelled.
One approach is to choose current patterns that are the right generalized singular
vectors of $Z - Z_{\text{calc}}$ with singular values bigger than an error threshold. The
generalized singular values are with respect to the norm $||\mathbf{I}||_Z := ||Z\mathbf{I}||$ on $S$
and are the extrema of the distinguishability defined as

$$||Z - Z_{\text{calc}}||_Z$$

for $\mathbf{I} \in S$. These excitation patterns are called “optimal current patterns” [44]
and can be calculated from an iterative procedure involving repeated measure-
ment. For circular disk with rotationally symmetric admittivity and equally
spaced identical electrodes the singular vectors will be discrete samples of a
Fourier basis and these trigonometric patterns are a common choice for multiple
drive systems using a circular array of electrodes.

Some of the references below are not needed, they are just taken from the
book chapter!
References


