MATH31011/MATH41011/MATH61011:
Fourier Analysis and Lebesgue Integration

Coursework

Please hand in your solutions to the Undergraduate Office Reception by 4.00pm on Thursday 6th November, with your name and course code on each sheet.

ANSWER BOTH QUESTIONS

1. Let X be a countable, infinite set. Prove that the set of all finite subsets of X is countable but that the set of all subsets of X is uncountable. (You may use any results established in the lectures or in the problem classes.)

2. Let D be the set of all $x \in [0,1]$ having a representation in the form

$$x = \sum_{i=1}^{\infty} \frac{a_i}{4^i}$$

where each a_i is either 0 or 3.

Give a geometrical description of D in the form

$$D = \bigcap_{n=0}^{\infty} D_n$$

where each D_n is a union of finitely many closed intervals. Hence show that D is a null set.

State (with brief justification) which of the following properties D has:

(a) D is countable;
(b) D is Borel;
(c) D is measurable;
(d) D is open;
(e) D is closed.