MATH43051/63051 Second midterm exam

Work to be handed in by 3 December 2014

Let $\mathfrak{A}$ denote the additive group of rational numbers, i.e. the structure $\langle \mathbb{Q}; +; 0 \rangle$. Let L be the language of $\mathfrak{A}$ and let T be the complete theory of $\mathfrak{A}$ (i.e. T consists precisely of those L-sentences ϕ such that $\mathfrak{A} \models \phi$).

By considering automorphisms of $\mathfrak{A}$ prove that every formula in $F_1(L)$ is $E_1(T)$-equivalent to exactly one of the four formulas

$$v_1 \doteq v_1, \ \ v_1 \doteq c, \ \ \neg v_1 \doteq c, \ \ \neg v_1 \doteq v_1.$$

(where c denotes the constant symbol of L, interpreted by 0 in $\mathfrak{A}$).
[7 marks]

Show that there are infinitely many $E_2(T)$-equivalence classes of formulas in $F_2(T)$.
[3 marks]