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www.ma.umist.ac.uk/avb/wherefrom.html
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Research Globally – Teach Locally

Where will the next generation of UK mathematicians come from? One of the first events hosted by the new Manchester Institute for Mathematical Sciences was the meeting under this heading held on 18-19 March 2005. The participants, widely recognised experts in mathematics and mathematical education, represented all principal constituencies of the education system. The brief of the Meeting was to look at the current state of mathematical education in the UK in its entirety, from pre-GCSE level to universities to graduate studies and teacher training.

The preliminary report produced by the Meeting (see http://www.ma.umist.ac.uk/avb/wherefrom.html) is based on the detailed (and sometimes technical) analysis of the current situation. But the nature of its findings made the report sufficiently quotable to generate unusually engaged responses in the media. Here is one example:

> "Maths teaching in schools and universities has entered "a spiral of decline" and the Government has failed to grasp the nature of the crisis, leading mathematicians said in a report yesterday. They said the performance of more able pupils had collapsed; the numbers taking A-level maths were falling dramatically; those with top grades were "increasingly innumerate and even ineducable"; the shortage of qualified maths teachers had reached "dangerous" levels; national test results were grossly inflated; and postgraduates with a PhD in maths from a British university were now "largely unemployable" in British universities." (The Daily Telegraph, 28 June 2005).

I think that it might be useful, without repeating the Report, to bring in my personal perspective as a co-organiser of the Meeting. Why did I feel that the alarming questions had to be asked and the meeting was necessary?

Our civilisation owes its existence to the eternal cycle of reproduction: Autumns' harvest is the next Spring's seed; a disciple steps in for his teacher. In this country, this natural cycle of reproduction of mathematics as a cultural system and a professional community is broken. This can be seen (and the Report confirms that) at every level of mathematical education. A simple question to those readers who work in universities' mathematics departments: can you claim that a PhD from your university warrants that your postgraduates can compete on equal with overseas candidates for a vacant Lectureship in your own Department?

Of course, the problem of recruitment of research mathematicians is easily solvable by bringing in fresh talent from overseas. The world mathematical community – and British mathematics – greatly benefits from the increasing mobility of researchers. Any positive discrimination or preferential treatment of British candidates – even if it was legal – would go against the academic ethos and undermine the gold standard of research. We should avoid the "League of Gentlemen" trap and not become "local universities – for local people".

But how to set the balance right? Paraphrasing the environmentalists' famous slogan, I propose the following simple principle:

Research globally, teach locally.

Mathematical research is one of the most globalised activities, and internationally acceptable standards of quality are the only ones that matter. But teaching mathematics is, by its very nature, local. Mathematical abilities in a child show themselves early, but are very fragile and have to be supported and nurtured for years. It cannot be done without mathematically competent and sensitive teachers at

school, without an exciting and challenging curriculum which stretches even the most able students, without a range of extracurricular activities or good mathematical books. The crucial stages of mathematical education take place in a local school and in the native tongue – frequently even in a local dialect.

I quote a part of the report which did not find its way in the newpapers:

> "Mathematical abilities are like musical abilities: in their developed form they appear highly specific, but are in fact quintessentially human, and so are widely spread in all social and ethnic groups in the population at large. As with music, mathematics has a profound and lasting educational impact – even where someone no longer uses their mathematical training in later life. Like music, success in mathematics depends on systematic, cumulative learning; and each new skill needs to be built on solid foundations, which need to be carefully laid at earlier stages. Though mathematics is often thought to be a "cold" subject, this is a profound misunderstanding; like music, it involves a high level of motivation and emotional involvement on the part of the learner. Understanding is of course vital; but it is a profound pedagogical mistake to think that teachers should minimise "difficulties" by lowering expectations."

Mathematics starts in childhood. Naturally, the school is the key component of the mathematical education system. For mathematical education to be inspiring and inclusive, teachers should preferably be recruited from university graduates who themselves were among the better and more enthusiastic students at school. Unfortunately, this natural cycle of reproduction of excellence is broken. The sad reality of modern school (and I was astonished to hear this in the Meeting!) is that weaker mathematics teachers cannot offer intellectual guidance to brighter students and suppress them, acting in the fear that an inquisitive student undermines a teacher's authority.

The Meeting proposed a very simple general recommendation for reversing the spiral of decline: "The educational system must address the needs, and cultivate the aspirations, of more able students (say the top 25%)", in the expectation that some of these students will find a career in mathematically intensive sectors of economy. Crucially, this will help to attract the best students to careers in mathematical education – both at the school and university level.

Although the main concern is about schools and teacher training, the university mathematicians have their fair share of work. The current approach to mathematics teaching, both at the school and the university level, is based on a very narrow understanding of the nature of mathematical skills. Teaching is reduced to coaching for (almost entirely written) examinations. Students (both at school and university level) are not exposed to two important mathematical activities: finding flaws in other people's arguments and conveying to others their own mathematical thinking. The damaging consequences of the poor mental diet become apparent at the level of teacher training. An effective mathematics teacher is a *diagnostician* and *communicator*: he or she *sees* student's error or difficulty, *understands* its underlying causes and *talks* to the student using an accessible mathematical language. The analytical and communicative skills required from a teacher are a part of a normal toolbox of a mathematician – but we do not give our students all the tools. We have a paradoxical situation when even reasonably good mathematics graduates are unprepared *mathematically* for a fruitful career in teaching. Indeed, students start *talking* about mathematics only at the graduate level. I knew university departments where PhD students had their own semi-secret graduate seminars and did not allow "grown-ups" to be present – a clear evidence of our failure as teachers.

I hope that the Meeting will start a serious and professional discussion of the problems of mathematical education at the university level. The Meeting made a number of suggestions which will be published in the full version of the Discussion Document (the Preliminary Report was mostly concerned with the school mathematics, as the most urgent task). Here, I do not want to pre-empt the document and refrain from any specific recommendations.

Alexandre Borovik, borovik@manchester.ac.uk