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A PhD project: "Development of group theory in the language of internal set theory"

The *internal set theory* proposed by Nelson [1] blurs the difference between finite and infinite sets in a very simple, effective and controlled way. To give you some taste and demonstrate the power of the new approach, I quote Alain Robert who starts his book on applications of internal set theory [2] with a reference to Leonard Euler:

> Here is how [Euler] deduces the expansion of the cosine function [...] He starts from the Moivre formula
>
> $$\begin{aligned}\cos nz &= \frac{1}{2}\left[(\cos z + i\sin z)^n + (\cos z - i\sin z)^n\right] \\ &= \cos^n z - \frac{n(n-1)}{1\cdot 2}\cos^{n-2} z\sin^2 z \\ &\qquad + \frac{n(n-1)(n-2)(n-3)}{1\cdot 2\cdot 3\cdot 4}\cos^{n-4} z\sin^4 z + \cdots\end{aligned}$$
>
> and writes [...]
>
> *sit arcus z infinite parvus; erit* $\cos\cdot z = 1$, $\sin\cdot z = z$; *sit autem n numerus infinite magnus, ut sit arcus nz finitae magnitudinis, puta $nz = v$*
>
> $$\cos\cdot v = 1 - \frac{v^2}{2!} + \frac{v^4}{4!} \text{—etc.}$$

In the context of internal set theory, this argument makes perfect sense. I am now in a position to explain that the proposed topic of a PhD study is a systematic development of the theory of finite and *pseudofinite* groups in the language of the internal set theory. This is motivated by problems in a branch of computational group theory, the so-called *black box recognition of finite groups*. Its typical object is a group generated by several matrices of large size, say, 100×100, over a finite field. Individual elements of such a group can be easily manipulated by a computer; however, the size of the whole group is astronomical, and arguments leading to identification of the structure of the group are being *de facto* carried out in an infinite object. The internal set theory provides tools that allow us to deal with finite objects and numbers that are, in effect, infinite. This is an exciting, unusual, but, as a look at Robert's book shows, accessible topic for study.

Prerequisites for the project: university level courses in algebra. Some knowledge of mathematical logic is desirable.

Alexandre Borovik

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[1] E. Nelson, Internal set theory: A new approach to nonstandard analysis, Bull. Amer. Math. Soc. 83 (1977), 1165–1198.

[2] A. M. Robert, **Nonstandard Analysis.** Dover Publications, 2003. ISBN0-486-43279-3.