A view from a mathematics teacher in a sixth-form college

Students see mathematics as a set of procedures to be followed. Their mathematical writing consists of instructions to do things and answers. Mathematics is about getting answers. Learning is about finding out how to get marks in examinations. Examinations test techniques in single-stage or short chains of reasoning. Once you know enough to get a good mark in one examination you rush on to the next. The short term is what matters, teachers and schools are only as good as their next set of examination results. There are easier options than mathematics.

I am not sure that what the top 20% need is that different from what the rest need—good teaching. This needs to start early, habits of thought are formed young and are hard to change. Moves to functional mathematics and a double award GCSE provide an opportunity for change which we must seize. We must not take the easy way out (which is far easier to deliver) of more content rather than more understanding. All phases from primary to higher education needs to see themselves as part of a common process and communication needs to happen in both directions.

Learners need to see mathematics as a form of reasoned discourse based on careful thought where distinctions are observed and conclusions are justified. Learners need to be able to construct sound arguments and apply techniques accurately. Learners need to be aware when particular actions are appropriate and when not.

To really understand a definition you need to be able to construct examples and counterexamples, this idea is rarely carried over into how students are taught. When learners acquire a new procedure they need to become aware of when it is appropriate and when not. This requires a more reflective approach than is often present now, where concentration is on implementing the procedure and little attention is given to scope and validity. Learners need an awareness of areas of mathematics beyond that over which they have procedural command. If overwhelmingly your experience of algebra is to solve equations, then you will not distinguish which procedures are proper
to equations and not to manipulating expressions or inequalities. If overwhelmingly your experience of brackets is in multiplication then you will stumble readily into false linearity. Learners need tasks which require them to decide whether given techniques are appropriate - so they acquire the counterexamples and so get some boundaries to the scope of the techniques they are learning.

Examinations and teachers need to value the quality of learners’ reasoning and its expression - marks must be given for it; examinations must be valid as well as consistent. We must cultivate precision of thought in learners so they make careful distinctions. Learners need to develop accuracy and fluency in arithmetical and algebraic processes, but what hope have they when they enter the sixth form unable reliably to match and distinguish among

\[ x + 2, \ 2 + x, \ x - 2, \ 2 - x, \ 2x, \ 2 \times x, \ x \times 2, \ x^2, \ 2^x, \ \frac{1}{2} x, \ \frac{2}{x}, \ x \div 2? \]

Learners need to be more reflective and seek to achieve understanding; too often students having difficulties are told to practice a technique before having a sound appreciation of it.

Mathematics is about relationships, to which techniques can be applied. Learners need to experience multi-step problems. They need to realise the value of capturing the information given in statements of relationship, and to be able to do so. They need situations where the subsequent choice of techniques to manipulate and combine those statements is not obvious and well-flagged. They need to be prepared to try alternative approaches and see this as healthy.

The aims given above should being applied in appropriate ways throughout (at least) secondary schooling. Examinations and other outside factors do influence what happens in the classroom, but teachers have far more influence and power over students’ experience of learning mathematics. We must take responsibility and act. Nevertheless, structures could be more helpful.

There should be a single-award GCSE (containing Functional Mathematics) with a double award GCSE building on it with some additional content but with more emphasis on the qualities outlined above. GCE mathematics should be re-developed with single and double subject awards in a similar style. (The title Further Mathematics should go; it has too much emotional baggage and can too often hide a motley collection of applied units.)

Performance indicators are too short-term, GCSE teaching should be judged not just by GCSE performance, but also by GCE performance, so one can see whether good foundations have been laid. Similarly GCE teaching should be judged also by degree performance. To make this work information needs to flow back upstream; institutions must not hide behind data protection issues.

Peter Thomas
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