The majority of the students attempted all questions in section A and attempts at section B were pretty evenly distributed over the questions. B4 was probably the least popular. Section A was generally OK, but A3 seemed to cause some systematic confusion, perhaps because it was **not** the same setup as in question 9 on Example Sheet 2, although it may have appeared so at first glance.

Attempts at the section B questions were generally balanced, with most students getting between half and three-quarters of the marks on each question. Students were much better at making use of the formula sheet this year and the answers were generally of good quality containing clear logical sequences of steps. There were still a few students who did not always explain what they were doing or why a particular formula was true: the initial statement of a formula or equation should have some accompanying explanation; e.g. conservation of mass requires ...

A1 This question tested understanding of the concept of objectivity. If students wrote anything reasonable for the first part (and most did) then marks were awarded. The second part was less well answered. Most students correctly stated that acceleration is not an objective quantity, but this statement was not always justified and many forgot to relate this statement to the actual question which asked why Newton's second law (for a particle) is not objective.

A2 This question tested understanding of deriving conservation laws in general and part (i) was answered well. Marks were lost for not explaining the initial equation (in some cases) and not always justifying each step of the argument. In part (ii) very few students recognised that if $S$ is defined as the number of particles per unit volume per unit time then it must be specified whether the volume is the deformed or undeformed one.

A3 This question tested understanding of deformation, strain measures, deformed and undeformed metrics. The setup is somewhat different from question 9 on Example Sheet 2 where two successive stretches are applied (i.e. two relative deformations). Here, you are given the final radii, so these are two absolute deformations. An extremely common problem was that students did not understand how to relate the quantities $\lambda$ and $r_1$. It was stated in the question that $\lambda$ is a constant (so it can't be a function of $r$) to be determined for each deformation. For the first deformation, the unit sphere is deformed into a sphere of radius $r_1$ so if $R = \lambda r$, then $\lambda = r_1$, so that $R = r_1$ when $r = 1$.

(i) The majority of students used the correct methods here and recognised that the strain tensor is diagonal. The most common answer was

$$\gamma_{11}^{(1)} = \frac{1}{2}(\lambda^2 - 1), \quad \gamma_{22}^{(1)} = \frac{1}{2}r^2(\lambda^2 - 1), \gamma_{33}^{(1)} = \frac{1}{2}r^2 \sin^2\theta(\lambda^2 - 1),$$

which is correct if $\lambda = r_1$. A few students didn't obtain the $\sin\theta$ term and a number set $r = 1$, which only gives the tensor at the outer surface of the sphere. The metric tensor is a function of the Lagrangian coordinates, which is defined (and varies) throughout the sphere.

(ii) There were more problems with this question because students had got confused about what is going on. You can continue to use the undeformed coordinates as the Lagrangian coordinates in all configurations. The sphere has now been uniformly stretched to a radius $r_2$ so $R = r_2 r$ and the non-zero entries of the deformed metric tensor are

$$G_{11} = r_2^2, \quad G_{22} = r_2^2 r^2, \quad G_{33} = r_2^2 \sin^2\theta.$$

The reference metric tensor is that associated with the first deformed configuration

$$g_{11} = r_1^2, \quad g_{22} = r_1^2 r^2, \quad g_{33} = r_1^2 \sin^2\theta,$$

giving

$$\gamma_{11}^{(2)} = \frac{1}{2}(r_2^2 - r_1^2), \quad \gamma_{22}^{(2)} = \frac{1}{2}r^2(r_2^2 - r_1^2), \gamma_{33}^{(2)} = \frac{1}{2}r^2 \sin^2\theta(r_2^2 - r_1^2).$$

(iii) In this case the metric tensor $\gamma_{ij}$ is of the same form as $\gamma_{ij}^{(1)}$ but with $r_1$ replaced by $r_2$.

(iv) Given the correct metric tensors is it straightforward to see that $\gamma_{ij} = \gamma_{ij}^{(1)} + \gamma_{ij}^{(2)}$.

B4 This question is similar to 2014 B6, but with a different constitutive law.

(i) This was generally answered well and was mostly bookwork. You need to show that $\mathsf{D}$, $\widehat{\mathsf{T}}$ are objective and that the upper-convected derivative of an objective quantity remains objective. Marks were typically lost for not stating explicitly that scalar quantities are objective, or forgetting to show that all required quantities are objective.

(ii) This was a moderately involved calculation, but those that knew how to start mostly got to the correct answer:

$$T_{XX} = \frac{2\lambda_1\epsilon - 1 + \sqrt{(1-2\lambda_1\epsilon)^2 + 4\alpha\lambda_1\epsilon}}{2\alpha\lambda_1/\eta_p}, \quad T_{YY} = \frac{-(2\lambda_1\epsilon + 1) + \sqrt{(1+2\lambda_1\epsilon)^2 - 4\alpha\lambda_1\epsilon}}{2\alpha\lambda_1/\eta_p}.$$

(iii) Most students who got this far correctly deduced that the law is valid when $\alpha \neq 0$, but not when $\alpha = 0$.

B5 This question was answered well by most that attempted it and was probably the best answered question in section B.

(i) This was similar to A2 with the same methods and most students got to the correct solution. Again errors were lack of clarity in the derivation and failing to state the origin of the first equation that was written down.

(ii) Almost everybody showed the desired relationship, which follows straightforwardly from the Reynolds Transport Theorem and the mass conservation result from (i). The second part about the balance of linear momentum was less well attempted. Many students just didn't attempt this part at all. Those that did got to the required solution in the majority of cases, but only a few tried to give requested interpretation. The interpretation is that the newly created mass moves with the material velocity of the solid.

(iii) This was straightforward and many students were able to answer this part of the question without doing the other two. The most common problem was forgetting to square the stretches in the deformed metric tensor, which has the form

$$G_{11} = \lambda_1^2, \quad G_{22} = G_{33} = \lambda_2^2,$$

with all other terms equal to zero. Another problem was forgetting to include $\lambda_g^2$ in the undeformed metric tensor. If the Green–Lagrange strain tensor is zero then $\lambda_g^2 = \lambda_1^2 = \lambda_2^2$. A few students forgot to write explicitly that this condition ensured that the strain was zero.

B6 This was material that was almost entirely covered in lecture notes in Example 5. For this reason, most students that attempted this question did very well. The problems were typically lack of sufficient explanation of the steps taken, particularly in the use of the Clausius–Duhem inequality. A few students struggled with the integration in part (iii).

B7 Parts (i) and (ii) of this question were mostly fine and were where the majority of marks were obtained. A few students got confused between $g_{ij}$ and $g^{ij}$, which meant that factors of $r^2$ were in the wrong place (or missing), but almost everybody knew what to do and did it correctly.

Part (iii) was not well answered, which probably indicates a lack of confidence with actually solving differential equations (rather than deriving them). The approach is to set $\sqrt{I_3} = 1$, which everybody knew, to obtain the equation $R'RZ'/r = 1$, where $R(r)$ and $Z(z)$ and the prime denotes differentiation with respect to the appropriate variable. Note that $Z' \neq 0$, and then $R'R/r = 1/Z'$, so we can apply the standard separation of variables argument because the left-hand side is a function only of $r$, whereas the right-hand side is a function only of $z$.

It follows that for some constant $\lambda$,

$$R'R = r/C \quad \text{and} \quad Z' = \lambda.$$

Integrating gives

$$\frac{1}{2}R^2 = \frac{1}{2\lambda}r^2 + D, \quad \text{and} \quad Z = \lambda z + E,$$

for new constants $D$ and $E$, which we set to zero to suppress rigid body displacements. This yields the required solution

$$R = \frac{r}{\sqrt{\lambda}}, \quad Z = \lambda z.$$

The forms of the strain invariants can then be found by direct substitution.

In part (iv), the vast majority of students did not recall that for an incompressible material $P$ is an independent variable. The stress tensor follows from direct use of the formula given at the end of the exam, but keeping $P$ independent.