MATH35021: EXAMPLE SHEET[1] IX

1.) For a body in plane strain (parallel to $z = 0$) let C_{AB} be a part of its boundary curve in the xy-plane, as shown in Fig. 1. Show that the resultant force (F_x, F_y) and the moment about the origin $\mathbf{M}_0$

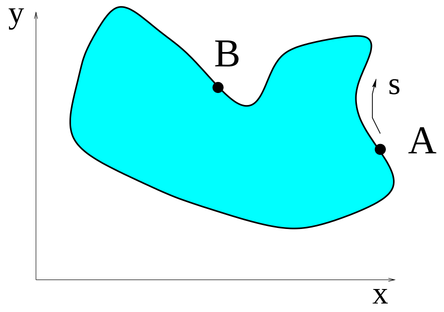

Figure 1: A rigid body in plane strain.

(all per unit length in the z-direction) of the tractions acting upon C_{AB} are given by

$$F_x = \left[\frac{\partial \phi}{\partial y}\right]_A^B, \qquad F_y = \left[-\frac{\partial \phi}{\partial x}\right]_A^B$$

and

$$\mathbf{M}_0 = \left[\phi - x\frac{\partial \phi}{\partial x} - y\frac{\partial \phi}{\partial y}\right]_A^B \mathbf{e}_z.$$

[Hint: Use integration by parts when determining $\mathbf{M}_0$].

2.) **a)** Show that

$$\nabla^2(\phi\psi) = \phi\nabla^2\psi + \psi\nabla^2\phi + 2\nabla\psi \cdot \nabla\phi.$$

b) Using the result from (a), show that the functions

$$F_1(x, y) = xH(x, y)$$

$$F_2(x, y) = yH(x, y)$$

$$F_3(x, y) = (x^2 + y^2)H(x, y)$$

are biharmonic if $H(x, y)$ is a harmonic function, i.e. $\nabla^2 H(x, y) = 0$.

3.) Show that the general solution to the axisymmetric biharmonic equation

$$\tilde{\nabla}^4\phi(r) = \phi_{,rrrr} + \frac{2}{r}\phi_{,rrr} - \frac{1}{r^2}\phi_{,rr} + \frac{1}{r^3}\phi_{,r} \qquad (1)$$

is given by

$$\phi(r) = A_0 + B_0 r^2 + C_0 \ln r + D_0 r^2 \ln r.$$

[Hint: (1) is a linear Euler equation].

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