MATH35021: EXAMPLE SHEET[1] V

1.) Show that in a homogeneous, isotropic, linearly elastic body the principal axes of the stress and strain tensors coincide. Derive the relation between principal strains and principal stresses.

2.) It was shown in the lecture that the stresses/tractions t_i acting at a certain point in the body, in a plane characterised by its unit normal vector n_j, are given by $t_i = \tau_{ij} n_j$. For a homogeneous, isotropic, linearly elastic solid, express the stresses/tractions in terms of the displacements u_i and thus show that

$$\mathbf{t} = \lambda\ \mathbf{n}\ \mathrm{div}\ \mathbf{u} + 2\mu(\mathbf{n}\cdot\nabla)\mathbf{u} + \mu\mathbf{n}\times \mathrm{curl}\ \ \mathbf{u}.$$

[Hints: (i) At some point you might find it useful to add and subtract $\mu\ \partial u_i/\partial x_j\ n_j$; (ii) Recall that $\omega_{ij} n_j$ can be written in symbolic form as $\boldsymbol{\omega}\times\mathbf{n}$ where $\boldsymbol{\omega} = 1/2$ curl $\mathbf{u}$.]

3.) Starting from the stress-strain relationships for a homogeneous, isotropic, linearly elastic body, $\tau_{ij} = \lambda\delta_{ij} e_{kk} + 2\mu e_{ij}$, derive the constitutive equation in terms of the two 'engineering constants' E and ν and thus show that:

$$\tau_{ij} = \frac{E}{1+\nu}\left(e_{ij} + \frac{\nu}{1-2\nu}\delta_{ij}\underbrace{e_{kk}}_{d}\right)$$

and

$$e_{ij} = \frac{1}{E}\left((1+\nu)\tau_{ij} - \nu\delta_{ij}\underbrace{\tau_{kk}}_{\theta}\right).$$

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