MATH35021: EXAMPLE SHEET[1] II

1.) A 2D body occupying the region $\{d : 0 \leq x_1 \leq 1, 0 \leq x_2 \leq 1\}$ is displaced by the following displacement field (see problem 3 on the last sheet) $u_1 = \epsilon(x_1 + 2x_2)$; $u_2 = \epsilon(3 + x_2)$ where $\epsilon \ll 1$.

a) Compute the extension $e_{\mathbf{n}_1}$ of a line element in the direction of the unit vector $\mathbf{n}_1 = (3/5, 4/5)^T$ and the change in the angle between $\mathbf{n}_1$ and $\mathbf{n}_2 = (-4/5, 3/5)^T$.

b) Find the principal axes and principal strains – i.e. the directions in which the deformation is purely extensional. Check the calculated principal strains by directly computing the extensions of line elements in these directions [since the strain is spatially constant, you can simply determine the lengths of finite line elements before and after the deformation]. Relate the principal axes to the sketched deformation of the body (from the last example sheet).

2.)

a) Show that $d = \text{div}\,\mathbf{u}$ and $\boldsymbol{\omega} = 1/2\ \text{curl}\,\mathbf{u}$.

b) Consider the rectangular parallelepiped, formed by the three infinitesimal vectors $(dx_1, 0, 0)^T, (0, dx_2, 0)^T, (0, 0, dx_3)^T$. Show that the fractional volume change of this infinitesimal volume element is given by d (as claimed in the lecture).

3.) Show that

$$\frac{\partial \omega_{ik}}{\partial x_j} = \frac{\partial e_{ij}}{\partial x_k} - \frac{\partial e_{kj}}{\partial x_i}.$$

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