Course 2Q1
Tutorial Sheet 1

The number in square brackets after the question number tells you the lecture(s) relevant to the particular example.

1. [1] Use linear interpolation to estimate sin(0.615), 0.615 measured in radians, using the pairs of known values for sin.

   (a) 0.0 and 1.0
   (b) 0.4 and 0.7
   (c) 0.55 and 0.65
   (d) 0.61 and 0.62

   For which pair of values is the most accurate approximation to sin(0.615) obtained?

2. [1] Find the linear function which is exact at $x = 4$ and $x = 9$ for the function $f(x) = \sqrt{x}$. Use interpolation to estimate $\sqrt{7}$ and extrapolation to estimate $\sqrt{11}$.

3. [1] Working in radians, given that $\tan(0) = 0$ and $\tan(1) = 1.5574$, use linear interpolation to estimate tan(0.76). Given, also, that $\tan(0.5) = 0.5463$, use quadratic interpolation to estimate tan(0.76). Find the error in the two approximations.

   Why would using tan(1.6) instead of tan(0.5) in the quadratic interpolation lead to poor results.

4. [2] Use the Trapezoidal rule, with 4 strips, to estimate the integral $\int_0^{\frac{\pi}{2}} \sin x \, dx$. Repeat the calculation for 8 strips and then 16 strips. Compare your answers with the exact value.

5. [2] Use Simpson's rule, with 4 strips, to estimate the integral $\int_0^1 e^x \, dx$. Repeat the calculation for 8 strips and then 16 strips. Compare your answers with the exact value.

6. [2] The following integrals cannot be performed algebraically. Estimate them using Simpson's rule.

   (a) $\int_0^{\pi} (\sin x)^{\frac{1}{2}} \, dx$
   (b) $\int_0^{\pi} e^{-x^2} \, dx$
   (c) $\int_1^2 x^{\ln x / \sin x} \, dx$
   (d) $\int_0^{\frac{1}{2}} \sqrt{\frac{1+x^2}{1+x^4}} \, dx$

7. [2] Let $f(x) = x^2 + e^{-1000(x-0.39)^2}$. Estimate $\int_{-1}^1 f(x) \, dx$, using Simpson's rule, with

   (a) 4 intervals (b) 8 intervals
   (c) 16 intervals (d) 32 intervals

   Sketch the graph of $f(x)$. Comment on your answers to parts (a)–(d).

8. [3] The equation $x^3 - 6x^2 + 11x - 5.8 = \ln x$ has several solutions.

(a) There is a solution between $x = 1$ and $x = 2$. Find it by the bisection method.

(b) There is a solution near $x = 3.3$. Find it by the Newton-Raphson method

(c) Find an interval containing a third solution.

9. [1,2,3] The root R of the function

$$F(x) = \cos(a(x+1)) - x$$

depends on the value of a, that is $R = R(a)$.

(a) Given that $R(0.2) = 0.9267$, $R(0.6) = 0.5822$ and $R(0.8) = 0.4207$ use quadratic interpolation to estimate $R(0.4)$.

(b) Using the estimate of $R(0.4)$ found in part (a) as a starting point, find $R(0.4)$ to 4 decimal places using the Newton Raphson method.

(c) Given also that $R(0) = 1$ and $R(1) = 0.2834$ use the trapezoidal rule in n strips, with $n = 5$, to estimate

$$I = \int_{a=0}^{1} g(a)\, \mathrm{d}a \quad \text{where} \quad g(a) = aR(a).$$

10. [3,4] The equation $\cosh(x) = 2$ has a solution between $x = 1$ and $x = 2$. Find it by

(a) the bisection method

(b) the Newton-Raphson method (with $x_0 = 1.5$)

(c) the method of fixed-point iteration with $x_0 = 1.5$. (Use the iteration $x_{i+1} = x_i - \alpha(\cosh(x_i) - 2)$).

Compare the speeds of the methods.

11. [3,4] The function $2^x - 5x + 1$ has a root between $x = 0$ and $x = 1$. Find it by

(a) the bisection method

(b) fixed point iteration (with $x_0 = 0.1$ and iterating function $\phi(x) = \frac{1}{5}(2^x + 1)$).

Compare the speeds of the two methods.

12. [4] Compute $\sqrt[3]{5}$ to four decimal places using the method of fixed point iteration. (Justify your choice of iterating function).

13. [3,4] How many solutions of the equation $(x-4)^2 = \dfrac{1}{x^2}$ lie between $x = 3$ and $x = 5$? Find these to three decimal places.