# MATH60171 Mathematics and Computation

**Credit rating:** 15
**Level:** MSc/ Level 4
**Delivery:** Semester One
**Lecturers:** Andrea Schalk and Harold Simmons

## General Description

This course unit is concerned with two aspects of mathematical logic. On the one hand it provides an introduction to category theory. This is an area of mathematics that provides an abstract framework for modelling logics and has close connections to proof theory. On the other hand categories also provide models for various models of computation, and this describes the second topic under consideration. Ultimately a connection between these at first sight quite different areas is made.

## Aims

To describe some mathematics that provides a highly adaptable structured framework for modelling proofs or computations; to compare several different theories of computation.

## Learning Outcomes

At the end of this unit students will understand some ways in which the notions of computation can be formalized: have seen mathematical theories modelling various aspects of logic and computation; be able to solve problems in various mathematical areas, and write up their solutions; have carried out independent reading in the context of the above.

## Prerequisites

A good grounding in mathematics. Some familiarity with parts of mathematical logic in the broad will help but this is by no means essential. Although the topics treated have some connections with Computation, Computer Science, and Numerical Analysis, no familiarity with any of these is required.

## Content

The unit can potentially draw on four or five different topics together with an account of how these interact. A choice from the following list will be made to suit the interests of the participants. Lecture notes exist on all of the following topics, including a number of exercises. The lectures will concentrate on the key issues of an area, and the students will be required to read the notes independently, and to solve (some of) the provided exercises.

**Category theory:** Basics, functors and natural transformations, limits and colimits, cartesian closed categories.

**Various λ-calculi**: Untyped, simply typed, and typed specifically for the natural numbers. The syntax, reduction mechanism, and typing discipline of these. The simulation of natural number gadgetry within these systems.

**Forms of recursion**: The primitive recursive functions, above and below these. The extended Grzegorczyk hierarchy. Measures of complexity.

**While loop programs:** Basic constructs, denotational semantics, how to capture various recursion operators.

**Domain theory:** Recursion operators, fixed point characterization, the simple domains.

**Categorical semantics:** How the various syntactic calculi considered above can be modelled within appropriate categories of domains and other things.

**Teaching and Learning methods**:

22 lectures, 11 examples classes, and assigned reading.

**Learning Hours**

| **Activity** | **Hours** |
|---|---|
| Private study | 107 |
| Assignments | 10 |
| Staff/student contact | 33 |
| Total | 150 |

**Assessment**

| **Assessment Activity** | **Length** | **Weighted within unit** |
|---|---|---|
| Two take home assignments | | 20 |
| Examination | 2½ hours | 80 |

**Core Learning Material**:

A full set of notes for each of the sections will be available. There will also be a suggested selection of textbooks and references.