Answer ALL FIFTEEN questions

Electronic calculators and formula tables are not permitted.

No prepared notes of any kind are to be brought into the examination room.

This examination makes up 75% of the overall assessment for this course unit.
1. Sketch graphs of the following real-valued functions of $x$ satisfying
   
   (a) $f(x) = |x - 1|^2 - 1|$; \hspace{1cm} (b) $f(x) = 1 + \sqrt{|x + 2|}$. \hspace{1cm} [4]

2. Sketch in a diagram of the complex plane where complex values of $z$ satisfy $1 < |z + 1 + 2i| < 2$. \hspace{1cm} [2]

3. Express the following complex numbers in the standard form $x + iy$
   
   (a) $\frac{1 - i}{3 + i}$ \hspace{1cm} (b) $(1 + i)e^{\frac{4i\pi}{3} + 1}$. \hspace{1cm} [4]

4. A function $f$ is defined so that
   
   $$f(x) = 1 - \frac{3}{x^2}, \hspace{1cm} x > 0.$$ \hspace{1cm} (a) Find a formula for the inverse function $f^{-1}(x)$ and give its domain;
   (b) sketch the graphs of $f^{-1}(x)$ and $f(x)$. \hspace{1cm} [6]

5. By using the definition, find the derivative of the following function
   
   $$f(x) = \sqrt{1 + 2x}$$
   from first principles. \hspace{1cm} [4]

6. Find the limits
   
   (a) $\lim_{x \to 1} \frac{\ln x}{\sin(\pi x)}$; \hspace{1cm} (b) $\lim_{x \to \infty} x^\frac{1}{4}$. \hspace{1cm} [4]

7. Find the equation for the tangent line to the curve defined by
   
   $$y^2 = x^3 + 3x^2$$
   at the point $(1, -2)$. At what points does this curve have a horizontal tangent line? \hspace{1cm} [6]
8. Sketch the region enclosed by the given curves:

\[ y = \sin\left(\frac{\pi x}{2}\right), \quad y = x. \]

Then find the area of the region for \( x > 0 \). \[4\]

9. Evaluate the definite integrals

(a) \[ \int_0^1 y(y^2 + 1)^5 \, dy; \quad \text{(b) } \int_0^4 |\sqrt{x} - 1| \, dx. \] \[6\]

10. Find the area of the region enclosed by the curve \( r = 2 + \cos(2\theta) \). Sketch this curve. \[6\]

11. Find the equation of the plane that passes through the point \((-1, 2, 1)\) and contains the line of intersection of the planes defined as \( x + y - z = 2 \) and \( 2x - y + 3z = 1 \). \[7\]

12. Find the directional derivative of the function \( f(x, y) = \ln(x^2 + y^2) \) at the point \((2, 1)\) in the direction of the vector \( \vec{v} = (-1, 2) \). \[4\]

13. Evaluate the double integral

\[ \iint_D \frac{2y}{x^2 + 1} \, dA, \]

where \( D = \{(x, y) \mid 0 \leq x \leq 1, \, 0 \leq y \leq \sqrt{x}\} \). \[5\]

14. Find the linear approximation of the function

\[ f(x, y) = \ln(x - 3y) \]

at the point \((7, 2)\) and use it to estimate \( f(6.9, 2.06) \). \[6\]

15. Find the volume of the solid under the surface \( z = 2x + y^2 \) and above the region \( D \) bounded by curves \( x = y^2 \) and \( x = y^3 \). Sketch the region \( D \). \[7\]