Contingencies I, Feedback on the exam held on 1 June 2015

• Question 1

This question was generally answered very well. Many students got full marks and those that didn’t in most cases made a simple slip. An error that did crop up on a few occasions was in the calculation of the probability under CFM in b(ii). The CFM approximation only applies to part years. Some students wrote down \( \frac{4.6}{43} \) as being equal to \( \left( \frac{48}{43} \right)^{4.6} \) when it should be \( \frac{47}{43} \times \left( \frac{48}{47} \right)^{0.6} \).

• Question 2

This question was reasonably well done overall. Some students tried to use R to carry out the calculation which is not possible as the mortality table in the question is not available in R (unless you were to develop your own functions from scratch!). Instead the question expected the student to express the assurance/annuity as a summation (standard bookwork) and then calculate the answer.

The assurance was the sum of two terms covering the possibility of death either at the end of year one or at the end of year two. The annuity was the payment of 1 immediately, 2 on survival to the end of year one and 3 on survival for two years.

A few students mixed up assurances and annuities, for example by applying a probability of survival in the form \( t_p_x \) to an assurance benefit which pays out on death and requires probabilities in the form \( t_q_x \).

• Question 3

There was more of a mix of answers to this question. A number of students were unable to carry out the required integration.

For part a), the formula, \( q_x = \exp(- \int_0^t \mu_{x+s} ds) \) is in the formula sheet but some students were unable to apply that trying, instead, to integrate \( \frac{1}{120-x} \) rather than \( \frac{1}{120-(x+s)} \).

For part b), a lot of students got the basic integration formula correct but integrated over the range \([0, 120]\) rather than \([0, 120-x]\). Some also wrongly thought that the integration was over \( x \) rather than over \( t \).
• Question 4

a) This first part was done well. A relatively common mistake was to forget to use the select table.

b) Students found this part to be a bit harder. Some students chose to calculate the retrospective rather than prospective reserve which was fine but perhaps a bit harder to do. However, if the prospective reserve was calculated then there was no need to apply select mortality as the calculation is carried out at age 60, long after the select period has expired. Selection remains relevant to the retrospective reserve which uses functions at age 50. Also some students calculated an EPV at age 50 rather than at age 60. Finally, and this applied to some other questions too, there was sometimes not enough detail of the calculation written down to see where a wrong answer had come from. Ideally all the factors calculated by $R$ should have been shown to see whether there was an error with only one factor, a few factors or all factors. More marks were given if it could be seen that some factors were accessed correctly than if it could not be determined whether any factors were looked up properly.

c) This part proved difficult for students. The first part was bookwork and generally was answered well. Students appeared to find it difficult, in the course of the exam, to generate the plot of the Reserve - an iterative approach based on the recursive formula did the trick. A point to remember was that the survival benefit had to be taken into account at $t = 25$, which serves to make $25V = 0$. This carried over into part (iii) where some students, whilst realising that the reserve steadily increases each year (and getting marks for that), thought that the maximum value was at 25 years rather than the correct answer of 24 years.

d) Those that attempted this part were able, mostly, to answer this.

e) Again reasonably well done by those who tackled it. The most common mistake by those who had the basic idea right was in the calculation of the death probability. Some used a select mortality (which makes a big difference) when it should be the ultimate rate as we are now 9 years on from when selection occurred (the insured passed a medical). Others calculated the probability of dying over the 9 years from age 50 (sometimes 10 years from age 50) when, in fact consideration is of the probability of death between ages 59 and 60 only, i.e. the student needed to look up $q_{59}$. 

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• Question 5

a) This wasn’t answered correctly by many. The approximation simply assumes that deaths occur on average half way through the year.

b) This was reasonably well answered. The problems were often around making the correct adjustment for monthly payments - the adjustment isn’t simply $-\frac{11}{24}$ but $-\frac{11}{24}(1 - v^{20} p_{40})$.

Also some students struggled to get the adjustment for the missing first month of expense right. A final problem was that quite a few students got the treatment of expenses wrong in the principle of equivalence formula. The equation is:

$$\text{EPV premiums} = \text{EPV benefits} + \text{EPV expenses}$$

Some students added the EPV of expenses to the EPV of premiums.

c) This was a challenging question.

Some students expressed their answer as an EPV rather than a PV. The basic requirement was to be able to write down the PV of the benefits, expenses and premiums (for clarity it was sensible to think of these separately) and distinguish between the PV if $T_{40} \leq 20$ or $T_{40} > 20$. For example, for the benefits, the PV was equal to $20,000 v^{T_{40}}$ if $T_{40} \leq 20$ and zero if $T_{40} > 20$.

The expression for the PV of premiums when $T_{40} \leq 20$ was particularly hard in terms of getting the term of payment right. What was required is the payment of premiums until the end of the month prior to death. To calculate this the student needed to use a floor function.

Say $T_{40} = 6.54$. Then the last premium would be paid at 6.5 years. This can generally be calculated as $\lfloor 12 T_{40} \rfloor / 12$. So for example, if we substitute $T_{40} = 6.54$ into this term, we get $\lfloor 12 \times 6.54 \rfloor / 12 = \lfloor 78.48 \rfloor / 12 = 78 / 12 = 6.5$. As the premiums are paid monthly in advance, $1/12$ needs to be added to the term in order to get the right number of payments.

Also, for $T_{40} > 20$, the PV of premiums is $12 P_{\ddot{a}}^{(12)}$, not zero or some function dependent on $T_{40}$.

d) This was also hard.

The answer is that it is not possible to make a profit if death occurs before the end of the term. A profit arises in relation to those who survive and for whom no benefit is paid.
One approach to show this is to look at the lowest possible PV for the benefits and come up with a simple calculation of a value above which the PV of premiums can not lie and then compare the two.

The smallest PV of benefits is $20,000v^{20} = £7,538$. As premiums are paid for at most 20 years, the PV of premiums is less than $20 \times$ the annual premium (i.e less than £1,661). From this it can be seen that the maximum value of the PV of premiums is always less than the lowest value for the PV of benefits (adding in the PV of expenses only makes the losses larger so does not need to be looked at to answer this question).