General comments: The use of quantifiers was relatively good (despite comments below), and Section A and Question B8 were done well. I was also pleased to see people reconstructing proofs rather than simply regurgitating notes. There were lots of answers where the writer was obviously not thinking clearly, with statements which made no sense whatsoever. It’s important to be able to read your own work critically to check it makes sense. If you find yourself writing a lot of prose, and re-explaining yourself, then you’re probably on the wrong track and not making sense! It’s also clear there are some who have not engaged at all with the course.

Question A1: This caused few problems.

Question A2: Careful with quantifiers.

Question A3: Again many people run into problems being careless about quantifiers. A few people just regurgitated the definitions in the order in which they appear in the lecture notes, failing to notice that they are in a different order in the exam. For part (d), it’s important to say what \( A \) is. Just writing “\( < \)” is not enough.

Question A4: As always, many people used the wrong inductive assumption. E.g., “assume the statement is true for all \( k \)” or “assume the statement is true for some \( k \)”.

Question A5: Few problems here.

Question A6: The usual problem with quantifiers, either missing or ambiguous, e.g., “\( a, b \in S. \) Then \( \ast \) is commutative if \( a \ast b = b \ast a \).” Does this mean it must hold for all \( a \) and \( b \), or just for some previously defined ones?

Question B7(a): Parts (i) and (ii) were done well. In part (iii), many people showed that there could not be a unique identity element (not immediately the same as showing there is none). I was lenient here and allowed this, even though we hadn’t at that point established that the identity element is unique.

Question B7(b): This is bookwork. Remember the inverse must work on both sides.

Question B7(c): Part (i) done ok. Very few correct solutions to (ii).

Question B8(a): Nearly everyone got full marks in this part.

Question B8(b): Most people got one solution (24). Most of these also got all six solutions. Some people got \( x = 12 \) by mistake.


**Question B8(c):** I was pleased with this question as it really sorted out the people who knew what they were doing (not very many!) From part (a) you have the information $6 = 12 \times 102 - 7 \times 174$. Divide this by 6 to get $1 = 12 \times 17 - 7 \times 29$. So $12 \times 17 \equiv 1 \mod 29$. So $12^{-1} = 17$. [It’s just an annoying coincidence that $12 + 17 = 29$.]

Many people showed poor exam technique by spending vast lengths of time on this, despite only three marks being available. An examiner would never ask you to produce a 29 by 29 table, or even to calculate all the products of 12 modulo 29!

**Question B9(a):** This was not done well at all. With an understanding of the definitions this should be very straightforward, just by working methodically. Understanding of the definition of an equivalence class was poor, with some real nonsense written, e.g., “$x \in aRb$”, “$aRx$ commutes,” etc.

**Question B9(b):** Part (i) was another question which is straightforward if you understand the definitions. The necessary and sufficient condition is that each equivalence class has only one element (there are a number of ways of saying this). Very few people wrote down anything that made any sense. Part (ii) was also done badly. In this case you just have to write down the equivalence classes, e.g., \{0, 3, 6, 9\}, \{1, 4, 7, 10\} and \{2, 5, 8, 11\}.

**Question B10(a):** I was shocked how badly this was done. It’s bookwork, but very easy if you are methodical. Most people wrote nonsense and got 0 marks for this part.

**Question B10(b):** Many people got 6 out of 8 marks for this part. The other two marks were for dealing with the case $x = 2$, $y = 1$ (which most people forgot to do).

**Question B10(c):** This was done ok in general.

**Question B11(a):** Part (i) was done well, as was part (ii) usually. $A \setminus B = \{(n, 2n) : n \in \mathbb{Z} \setminus \{1\}\}$.

**Question B11(b):** Bookwork, and was largely done ok. Quite a few people forgot to take the minimal natural number greater than one with no prime divisor (the nub of the proof). Quite a few people presented it as a proof by induction, which is fine but quite a few stumbled in doing so. You cannot assume that every number has a prime factorisation - this is a consequence of the result you’re proving!

**Question B11(c):** This was done very badly. The idea is to reduce modulo 4 or 8, as in similar examples, but most people tried to treat it as a quadratic by making some assumption about $y$, so coming up with a proof that only worked in certain cases.

**Question B12:** I warned people many times during lectures that they would probably ignore the definitions in the “Counting” section, and this was indeed the case. Very few people got more three or four marks for this question, despite it being really very easy.
**Question B12(a):** The answer is not “A has cardinality $n$ if it has $n$ elements”! This statement is meaningless. The set $A$ has cardinality $n$ if there is a bijection $\mathbb{N}_n \to A$ (where $\mathbb{N}_n = \{1, \ldots, n\}$).

**Question B12(b):** I was not accepting $\frac{n!}{r!(n-r)!}$.

**Question B12(c):** This is from notes. Few few marks given for this question.

**Question B12(d):** Bookwork, done reasonably by those who attempted it.

**Question B12(e):** Similar to an exercise. Done ok, but quite a few people tried fruitlessly for an inductive argument.