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What is the probability of a set to have a given property?

The aim is to define what it means for a property A of sets y to have a probability of being true of a set in the set theoretic universe. Properties are identified with their extensions, so that A ranges over all proper and improper classes. In other words, we want to develop a theory of the probability of events of the form $A(y)$, where A is a class and the variable y is a *random variable*, ranging over all sets. I am interested in *uniform* probability distributions. That is, all singleton events in the sample space V should be equiprobable: the probability of $y = a$ should be the same for every set a. In addition, we want the resulting probability functions to be *regular*: only properties that do not hold of *any* set should get probability 0. Also, I would like the resulting functions to be *total*: every property should determine a *measurable* class. Of course this means that the sought-for probability functions cannot be Kolmogorov probability functions. They will turn out —on the construction that is proposed here— to be non-Archimedean class functions, and they will not satisfy σ-additivity but a generalised infinite additivity rule.