Weakly U-abundant semigroups

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Abstract

Let S be a regular semigroup and let $E(S)$ be the set of idempotents of S. Then, equipped with two quasi-orders, $E(S)$ forms a structure called a *regular biordered set*. Conversely, if E is a regular biordered set, then E is the set of idempotents of a regular semigroup. Much more than this, a well known result of Nambooripad [3] shows that the category of regular semigroups is equivalent to a category of inductive groupoids over regular biordered sets, thus establishing a correspondence between algebraic structures and ordered structures.

These ideas have been extended in many directions, for example to inverse semigroups and inductive groupoids over semilattices (the Ehresmann-Schein-Nambooripad (ESN) theorem), to Ehresmann semigroups and Ehresmann categories [2] and to concordant semigroups and inductive cancellative categories over regular biordered sets [1].

We will look at some recent developments. Let U be a regular biordered set. Weakly U-concordant semigroups are weakly U-abundant semigroups with (C) and U generating a regular subsemigroup whose set of idempotents is U. We build a correspondence between weakly U-concordant semigroups and certain categories with two quasi-orders, which are analogous to inductive groupoids. This both requires new techniques and puts the existing results into a general context.

References

[1] S. M. Armstrong, 'Structure of concordant semigroups', *J. Algebra* **118** (1988), 205-260.

[2] M. V. Lawson, 'Semigroups and ordered categories. I. The reduced case', *J. Algebra*, **141**(1991), 422-462.

[3] K. S. S. Nambooripad, 'Structure of regular semigroups I', *Mem. Amer. Math. Soc.*, **22**(1979), No. 224.