**The word problem for semigroups**

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**Abstract**

One of the classical problems in group theory is that of the word problem. Given a presentation (i.e. a set of generators and a set of defining relators) for a group G, we can ask whether or not a word in the generators represents the identity element of G. An alternative formulation is to think of the set W of words that represent the identity element of G and then ask whether or not a given word lies in W. This leads to some very interesting connections between group theory and formal language theory, asking (for example) which groups have their word problem in some specified class of formal languages.

As it stands, this idea does not generalise naturally to monoids and

semigroups. Whereas, in a group, knowing which words represent the

identity allows us to tell when a pair of words represent the same

element, this does not work in monoids (and is clearly meaningless in arbitrary semigroups). Duncan and Gilman have proposed an elegant

definition for the word problem of a semigroup S. Given a generating set A for S, we pick some new symbol # (i.e. some symbol not in A) and then consider the set of all words of the form u#rev(v) where u and v represent the same element of S (and where rev(v) denotes the reversal of the word v). This is a natural extension of the definition for groups since, given a family F of languages, the word problem of a group in the group sense lies in F if and only if the word problem of the group in the semigroup sense lies in F.

In this talk we will introduce these ideas and survey some of what is known about word problems of semigroups. We will not assume any prior knowledge of formal language theory.